

Vistula, IT Faculty, 2014

Algorithms and Complexity

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Content I

1. Introduction to algorithms and complexity
2. A case study: paths in triangle of numbers
3. Turing machine as a model of computations
4. Undecidable problems
5. Time and space complexity of algorithms
6. Arrays, tables and lists
7. Graphs and trees

Content II

8. Recursive algorithms
9. Sorting and merging algorithms
10. String and search algorithms
11. Nondeterministic Turing machine
12. NP-complete problems
13. Divide and conquer approach
14. Dynamic programming
15. Paradigms of programming

A first pass - roughly

Introduction to algorithms and complexity

- Definition of algorithm
- Features of algorithm
- Hilbert list and necessity of formalization
- Known models of computations
- Correctness and complexity
- Undecidable, intractable, and tractable
- Algorithms and programs

A case study: paths in pyramid of numbers

- A task – maximal sum on a path in pyramid
- Exhaustive search algorithm
- Programming and testing
- Storing intermediate sums algorithm
- Programming and testing
- Generating source data
- Running programs to estimate time

Turing machine as a model of computations

- Description of TM: tape and control head
- Rules of TM work
- Examples of TM
- TM representation: graph, table, instructions
- Thesis of Church
- TM encoding
- Universal TM as a prototype of computers

Undecidable problems

- Self-applicability of TM
- Proof that SATM problem is undecidable
- Reduction technique to prove undecidability
- TM halting problem
- List of undecidable problems
- How to do with undecidable problems
- Other models of computation

Time and space complexity of algorithms

- Time and space complexity of TM
- Time and space complexity of a program
- Best, worse, and average complexity
- Asymptotic estimations of complexity
- Linear and logarithmic scale
- Calculating time complexity
- Calculating space complexity

Arrays, tables, and lists

- One dimension array and matrix
- D-dimension arrays
- Tables and search
- One link lists
- Two link lists
- Basic operations
- Stacks and queues

Graphs and trees

- Concept of a graph
- Adjacency and incidence matrices, lists
- Examples of graph algorithms
- An exhaustive search tree
- Depth and width detours
- Binary tree to store data
- Numerical trees

Recursive algorithms

- Factorial and Fibonacci numbers
- Hanoi tower puzzle
- Tracing recursive calls
- Complex and mutual recursion
- Recursive algorithms for lists
- Recursive algorithms for trees

Sorting and merging algorithms

- Sorting via sequential minimums
- Estimation of complexity
- Merging two arrays
- Fast sorting
- Other sorting algorithms

String and search algorithms

- Comparing two strings
- Substring matching
- Linear search and binary search
- Hash tables
- Binary tree search
- Numerical tree search

Nondeterministic Turing machine

- Definition of NDTM
- Examples of NDTM
- P and NP problems
- Converting NDTM to TM
- Nondeterministic choice in algorithms

NP-complete problems

- Whether $P=NP$?
- Other complexity classes
- NP-completeness
- Cook theorem
- List of NP-complete problems
- How to do with NP-complete problems

Divide and conquer

- Divide-and-conquer principle
- Multiplication
- Sorting and merging arrays
- Matrix multiplication
- Branch and bound approach – to limit the number of variants
- Bellman-Ford algorithm
- Travelling salesman problem

Dynamic programming

- Principles of DP: store and reuse intermediate results
- Simplest example – calculating Fibonacci numbers
- Paths in a triangle of numbers
- Paths in directed acyclic graphs
- Knapsack problems
- Chain matrix multiplication

Paradigms of programming

- Imperative and declarative approaches
- Structural programming
- Object-oriented programming
- Functional programming
- Logical programming
- Data flow programming
- Parallel and distributed programming

Literature

- Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein, Introduction to Algorithms, The MIT Press
- Alfred V. Aho, John E. Hopcroft, Jeffrey D. Ullman, The Design and Analysis of Computer Algorithms, Addison-Wesley
- László Lovász, Complexity of Algorithms, Eötvös Loránd University
- Herbert S. Wilf, Algorithms and Complexity, A K Peters/CRC Press
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A second pass – thoroughly
by lectures