

Clans Aggregation for Verification of Networking Protocols on Parallel Architectures

Dmitry A. Zaitsev

*Information Technology Department
Odessa State Environmental University*

Odessa, Ukraine

daze@acm.org

<http://daze.ho.ua>

Tatiana R. Shmeleva

*Switching Systems Department
A.S.Popov Odessa National
Academy of Telecommunications*

Odessa, Ukraine

t.shmeleva@onat.edu.ua

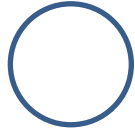
Alexander A. Kostikov

*Informatics and Engineering
Graphics Department
Donbass State Engineering Academy*

Kramatorsk, Ukraine

alexkst63@gmail.com

Elements of a Petri net



place



transition



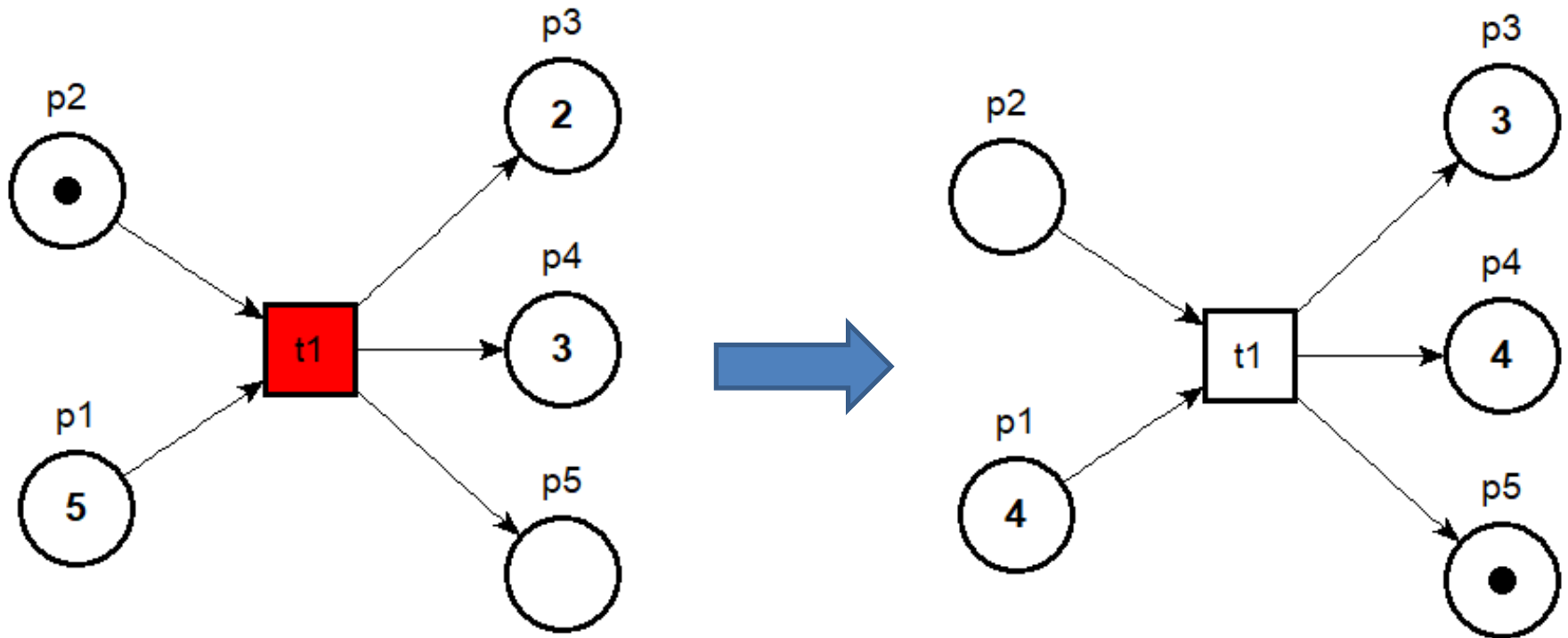
arc



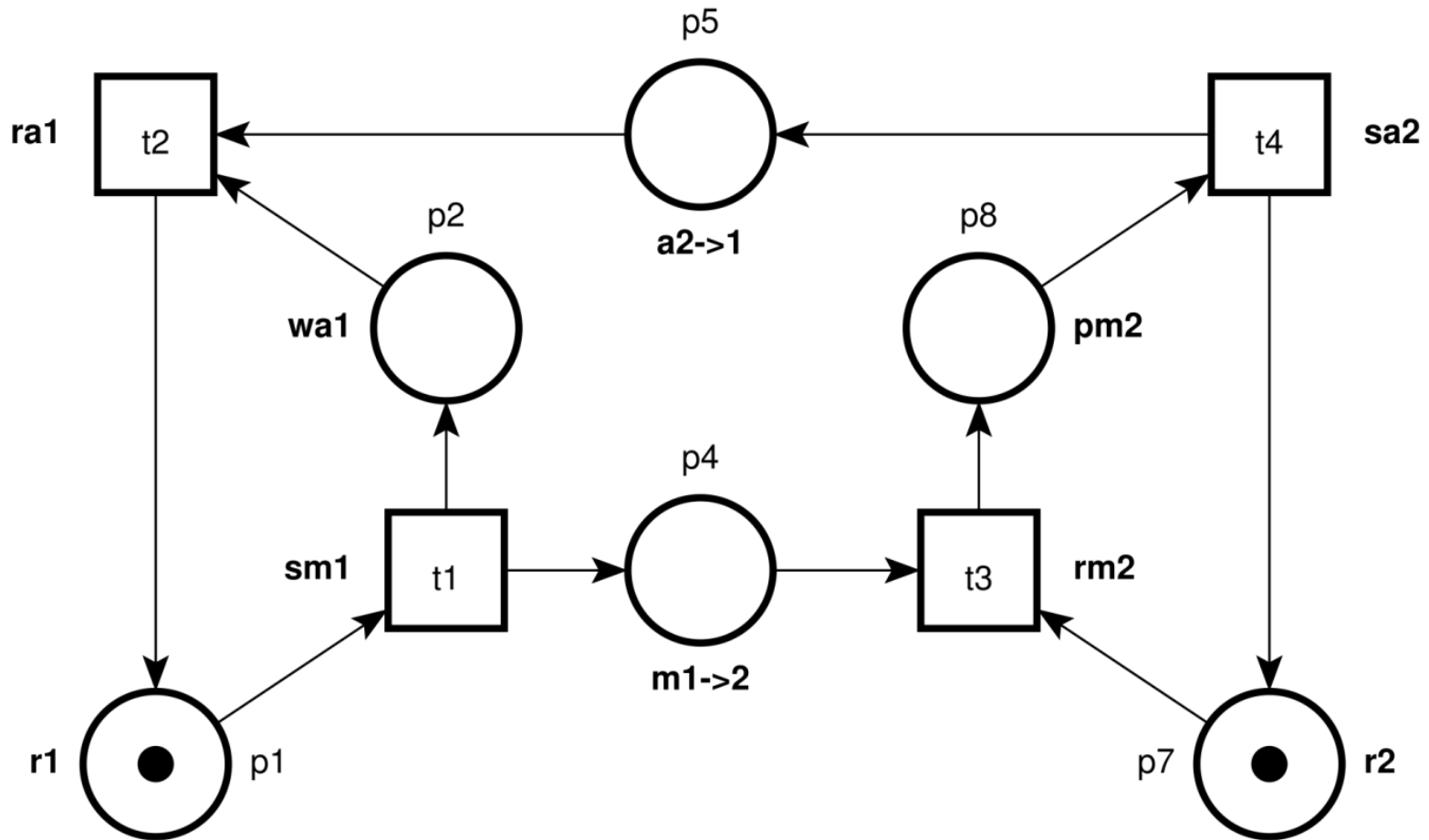
token



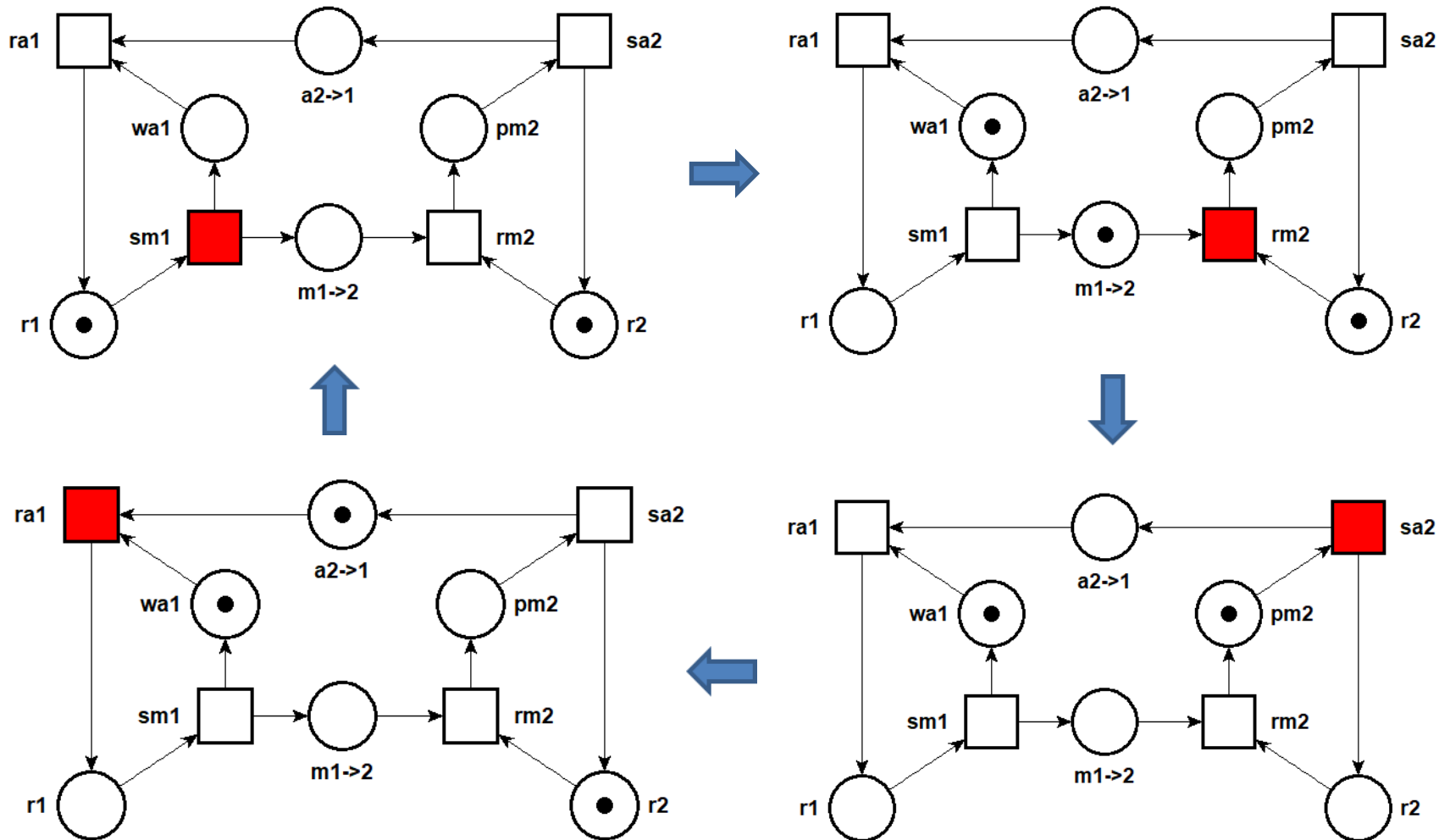
Petri net transition firing



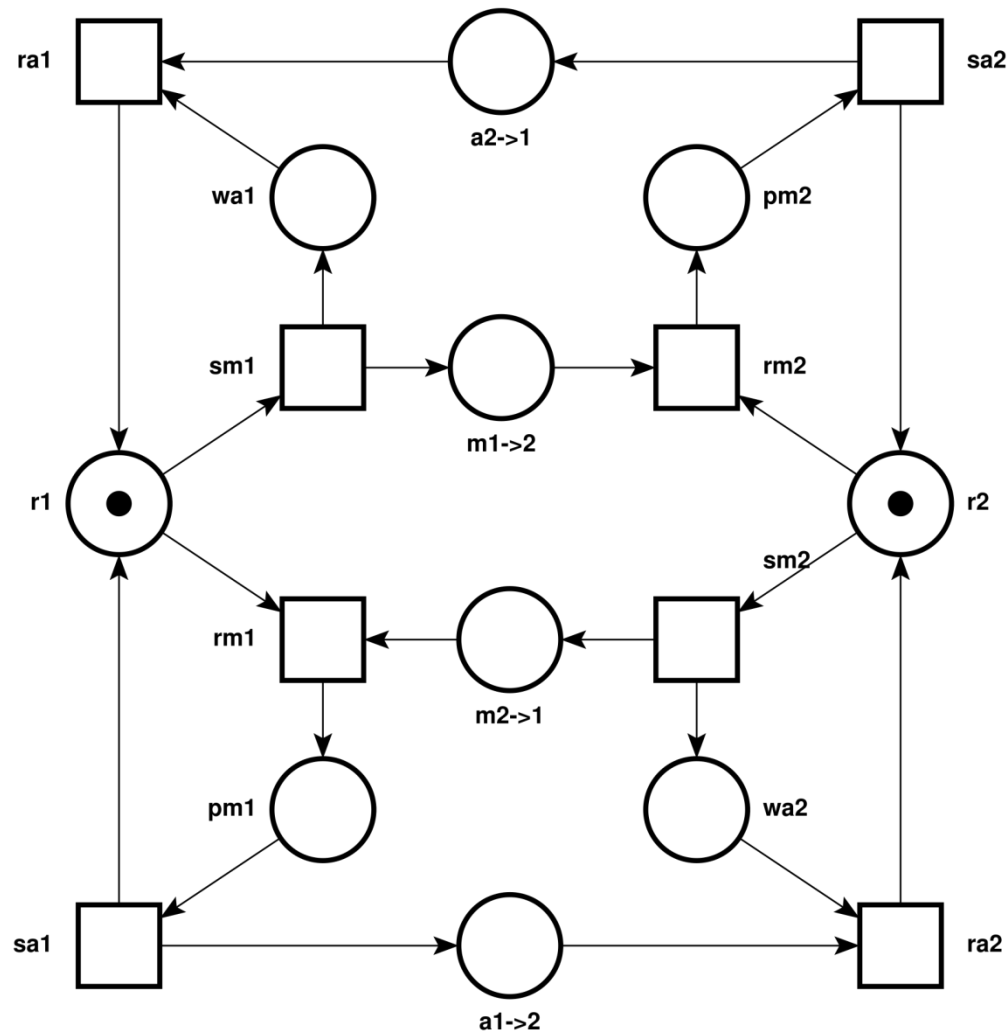
Protocol of one-way transmission with acknowledgements



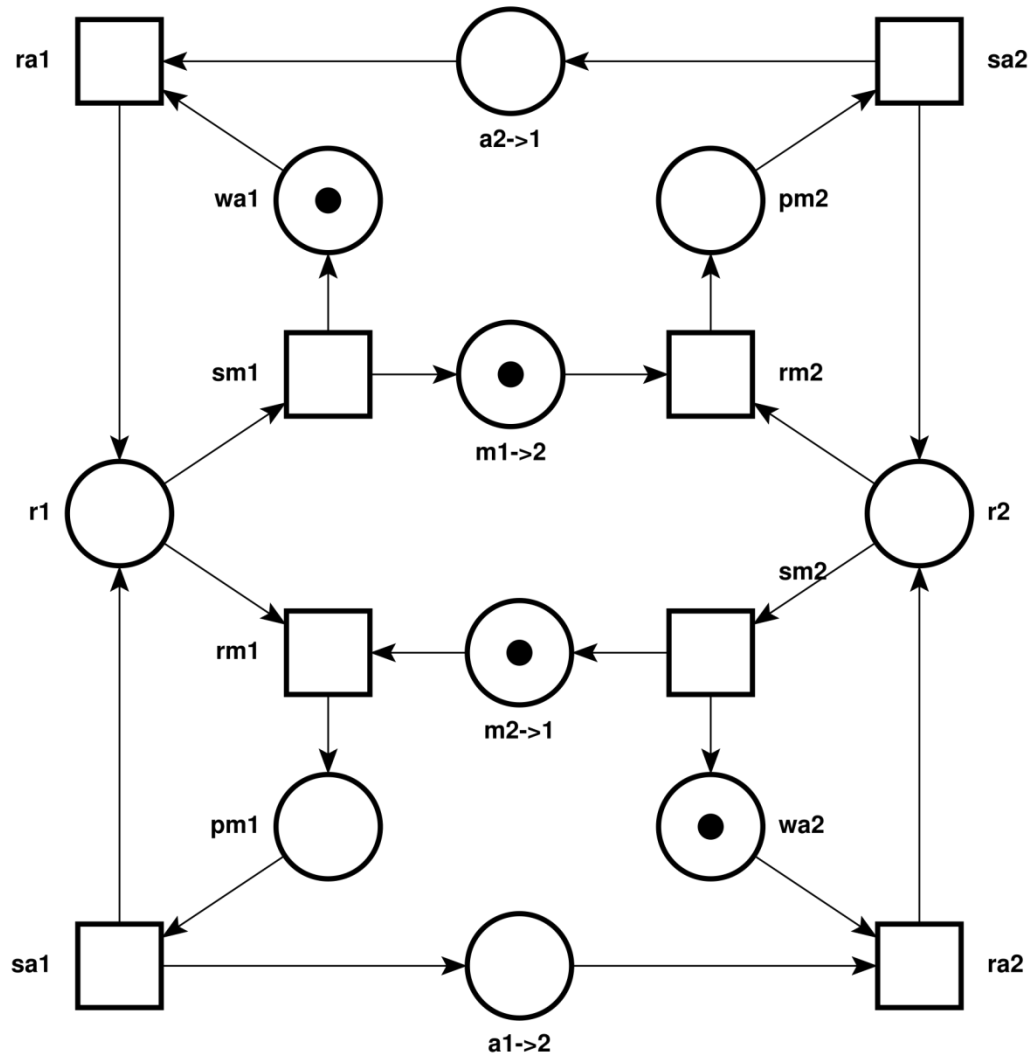
Petri net behavior



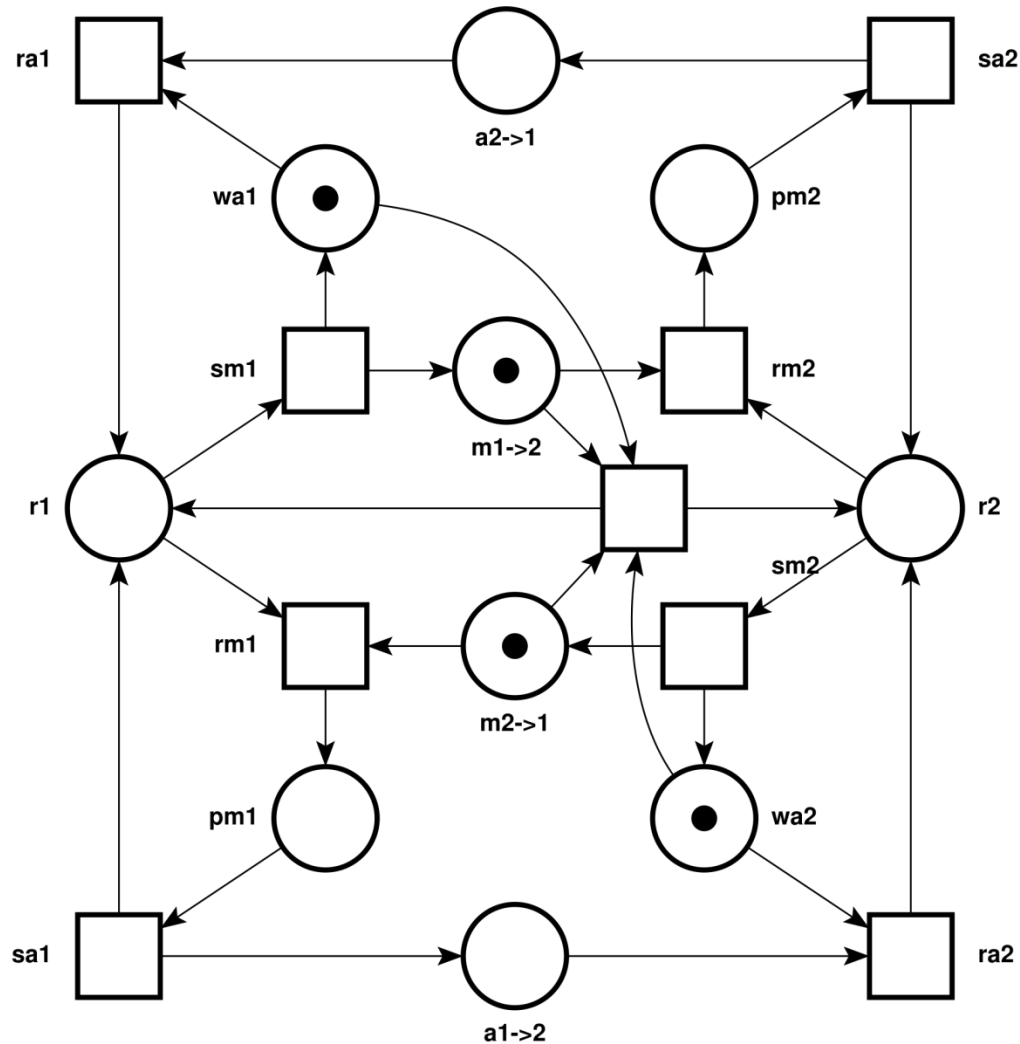
Protocol of two-way transmission with acknowledgements



Deadlock



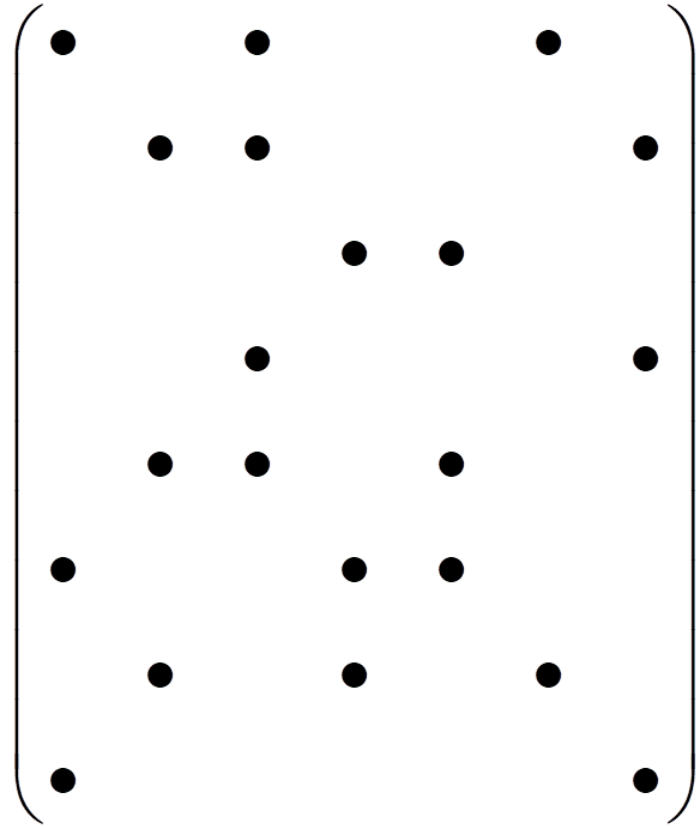
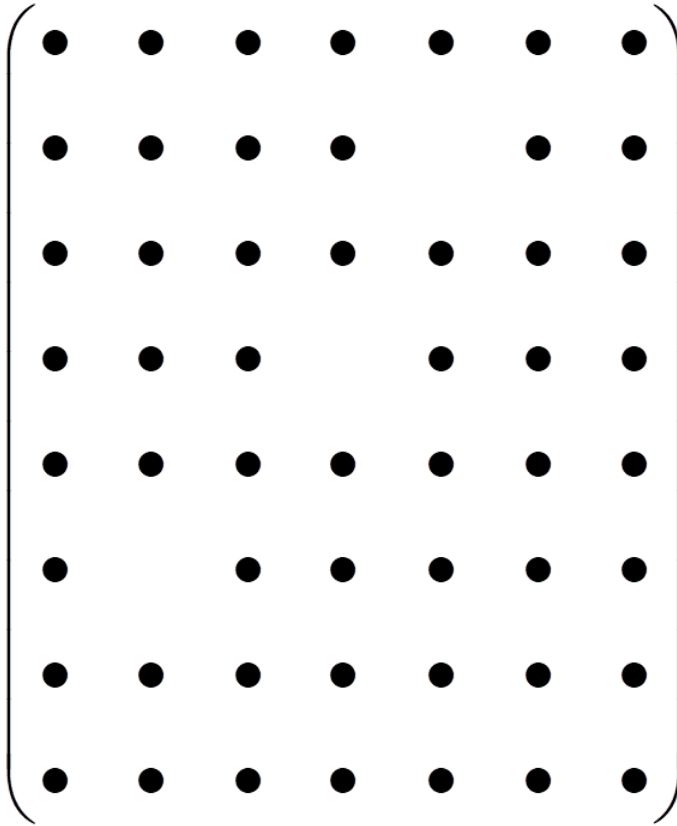
Modified protocol: collisions detection



Methods of Petri net analysis

- Reachability and coverability tree
- **Fundamental equation and linear invariants – solving linear systems over integer nonnegative numbers**
- Siphons and traps
- Reduction – transformations preserving properties
- **Decomposition – divide into parts**

Dense and sparse systems



Algebraic structure

Numbers	Example	Structure	Methods
Complex	-3,2+6,25i	field	a) reduction: LU, QR; 6) iteration methods
Real	0,25; -78,931		
Integer	-33; 0; 6	ring	Normal forms: Hermite, Smith
Nonnegative integer	0; 7; 55	monoid	Methods of Toudic (Silva) and Contejean

Real-life models

- Model Checking Contest – Petri net models <https://mcc.lip6.fr>
- Matrix Market - <https://math.nist.gov/MatrixMarket>
- The SuiteSparse Matrix Collection <https://sparse.tamu.edu>

Basic software

Structure \ Type	Dense	Sparse
Field	LAPACK	UMFPACK
Ring	4ti2	ParAd+4ti2
Monoid	4ti2	ParAd

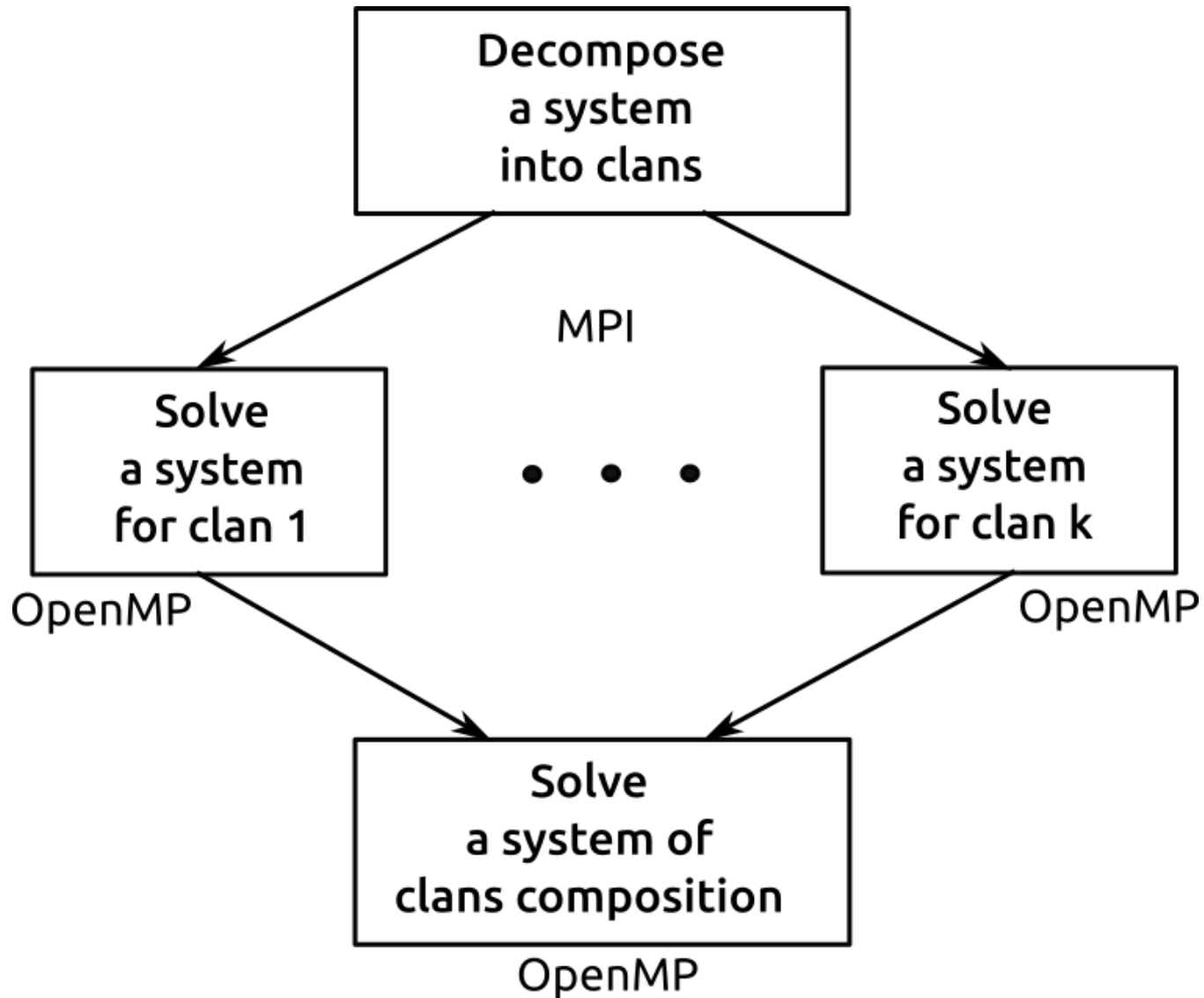
Software

- ***Deborah*** – decomposition into clans, 2004
- ***Adriana*** – solving a homogenous system via (a) simultaneous or (b) sequential composition of clans, 2005
- ***ParAd*** – solving a homogenous system via (a) simultaneous or (b) parallel-sequential composition of clans on *parallel architectures*, 2017

Zaitsev decomposition into clans

$$A = \left\| \begin{array}{ccccc} A^{0,1} & \widehat{A}^1 & 0 & 0 & 0 \\ A^{0,2} & 0 & \widehat{A}^2 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A^{0,k} & 0 & 0 & 0 & \widehat{A}^k \end{array} \right\|$$

Divide and sway

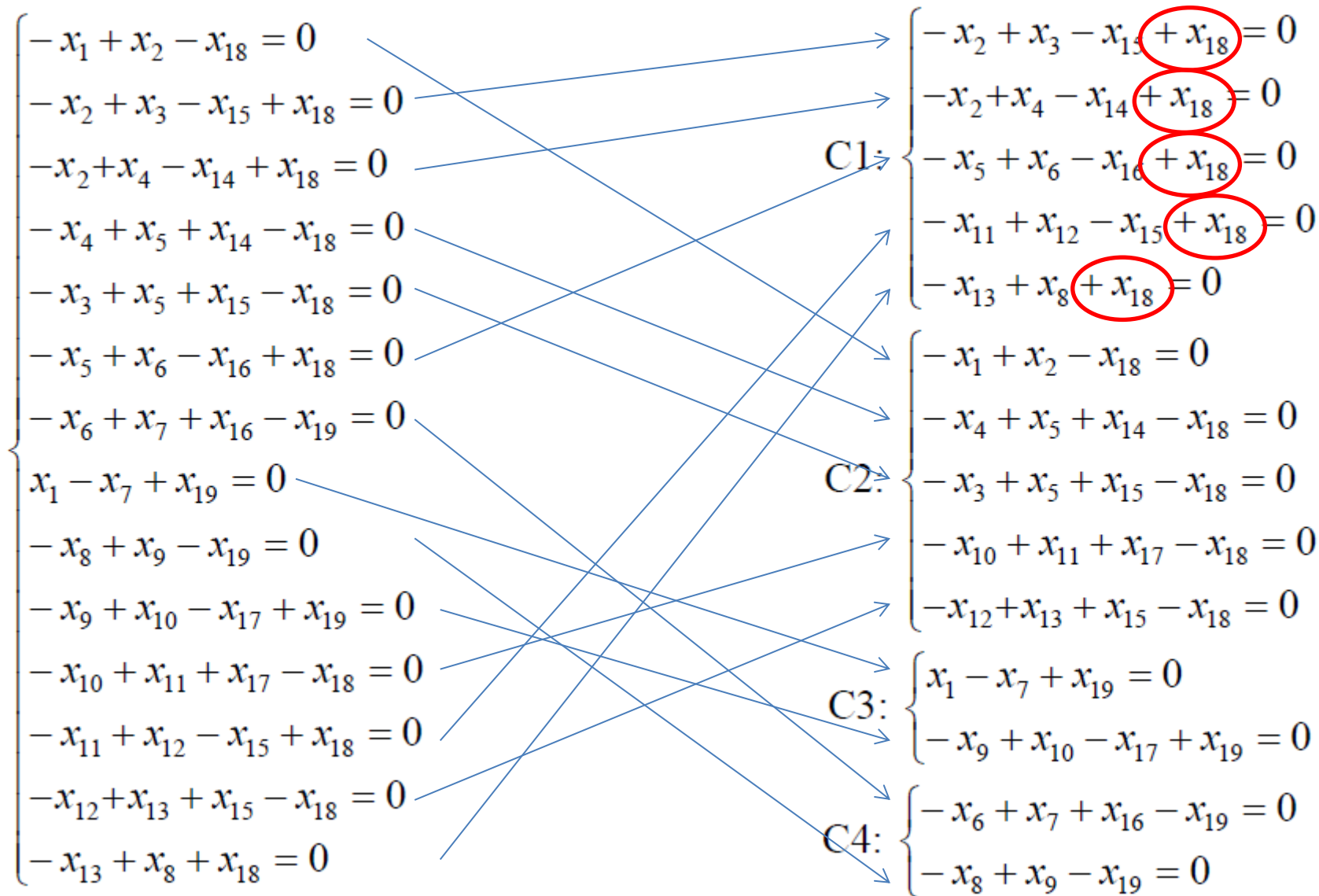


A Clan – transitive closure of nearness relation

$$\text{C1: } \left\{ \begin{array}{l} -x_2 + x_3 - x_{15} + \underline{x_{18}} = 0 \\ -x_2 + x_4 - x_{14} + \underline{x_{18}} = 0 \\ -x_5 + x_6 - x_{16} + \underline{x_{18}} = 0 \\ -x_{11} + x_{12} - x_{15} + \underline{x_{18}} = 0 \\ -x_{13} + x_8 + \underline{x_{18}} = 0 \end{array} \right.$$

Two equations are ***near*** if they contain the same variable having coefficients of the same sign

Decomposition into clans



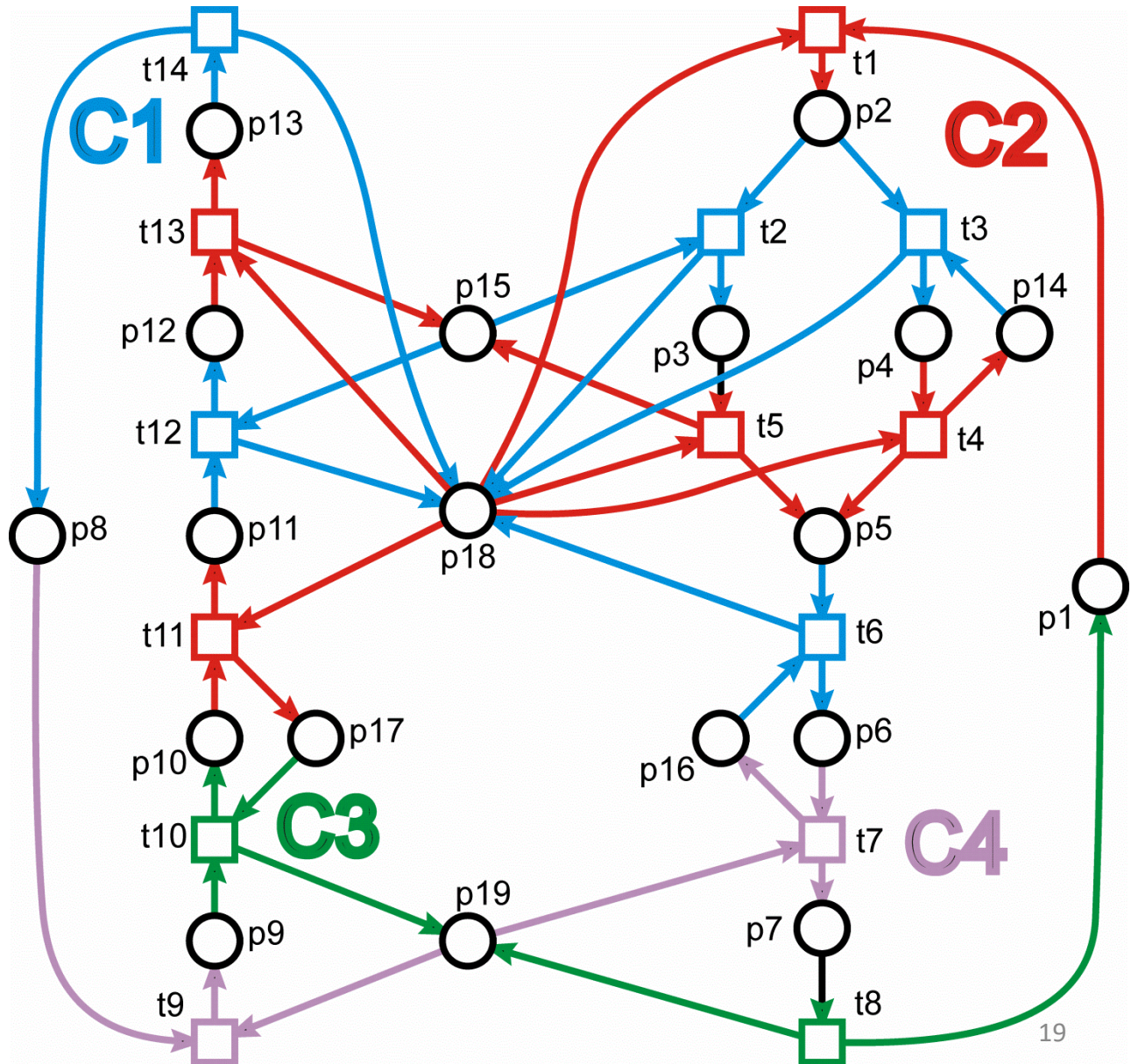
Systems and Directed bipartite graphs

Equation –
transition (rectangle)

Variable –
place (circle)

Positive sign –
incoming arc of a
place

Negative sign –
outgoing arc of
a place



Decomposition graph

$$C1: \begin{cases} -x_2 + x_3 - x_{15} + x_{18} = 0 \\ -x_2 + x_4 - x_{14} + x_{18} = 0 \\ -x_5 + x_6 - x_{16} + x_{18} = 0 \\ -x_{11} + x_{12} - x_{15} + x_{18} = 0 \\ -x_{13} + x_8 + x_{18} = 0 \end{cases}$$

$$C2: \begin{cases} -x_1 + x_2 - x_{18} = 0 \\ -x_4 + x_5 + x_{14} - x_{18} = 0 \\ -x_3 + x_5 + x_{15} - x_{18} = 0 \\ -x_{10} + x_{11} + x_{17} - x_{18} = 0 \\ -x_{12} + x_{13} + x_{15} - x_{18} = 0 \end{cases}$$

$$C3: \begin{cases} x_1 - x_7 + x_{19} = 0 \\ -x_9 + x_{10} - x_{17} + x_{19} = 0 \end{cases}$$

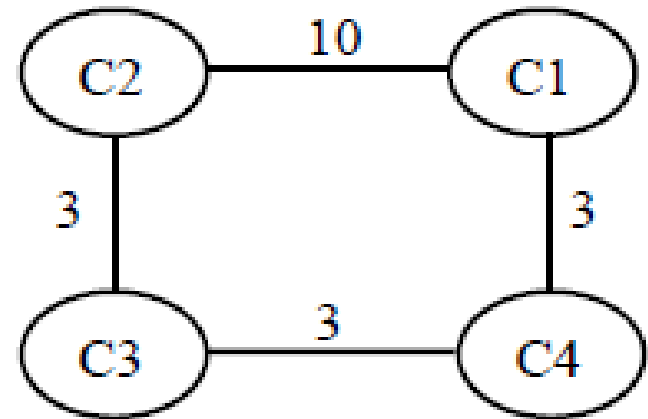
$$C4: \begin{cases} -x_6 + x_7 + x_{16} - x_{19} = 0 \\ -x_8 + x_9 - x_{19} = 0 \end{cases}$$

$x_1, x_2, x_3, x_4, x_5, x_{11}, x_{12}, x_{13}, x_{15}, x_{18}$

x_1, x_{10}, x_{17}

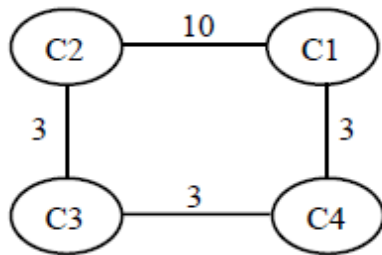
x_6, x_8, x_{16}

x_7, x_9, x_{19}

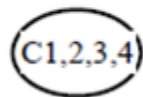


Collapse of decomposition graph

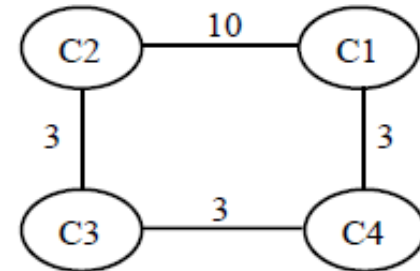
I.



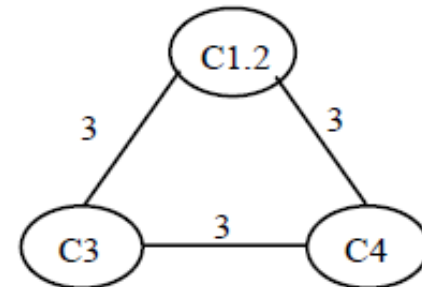
$\Downarrow 19$



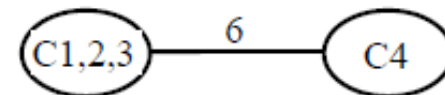
II.



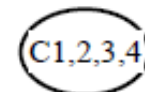
$\Downarrow 10$



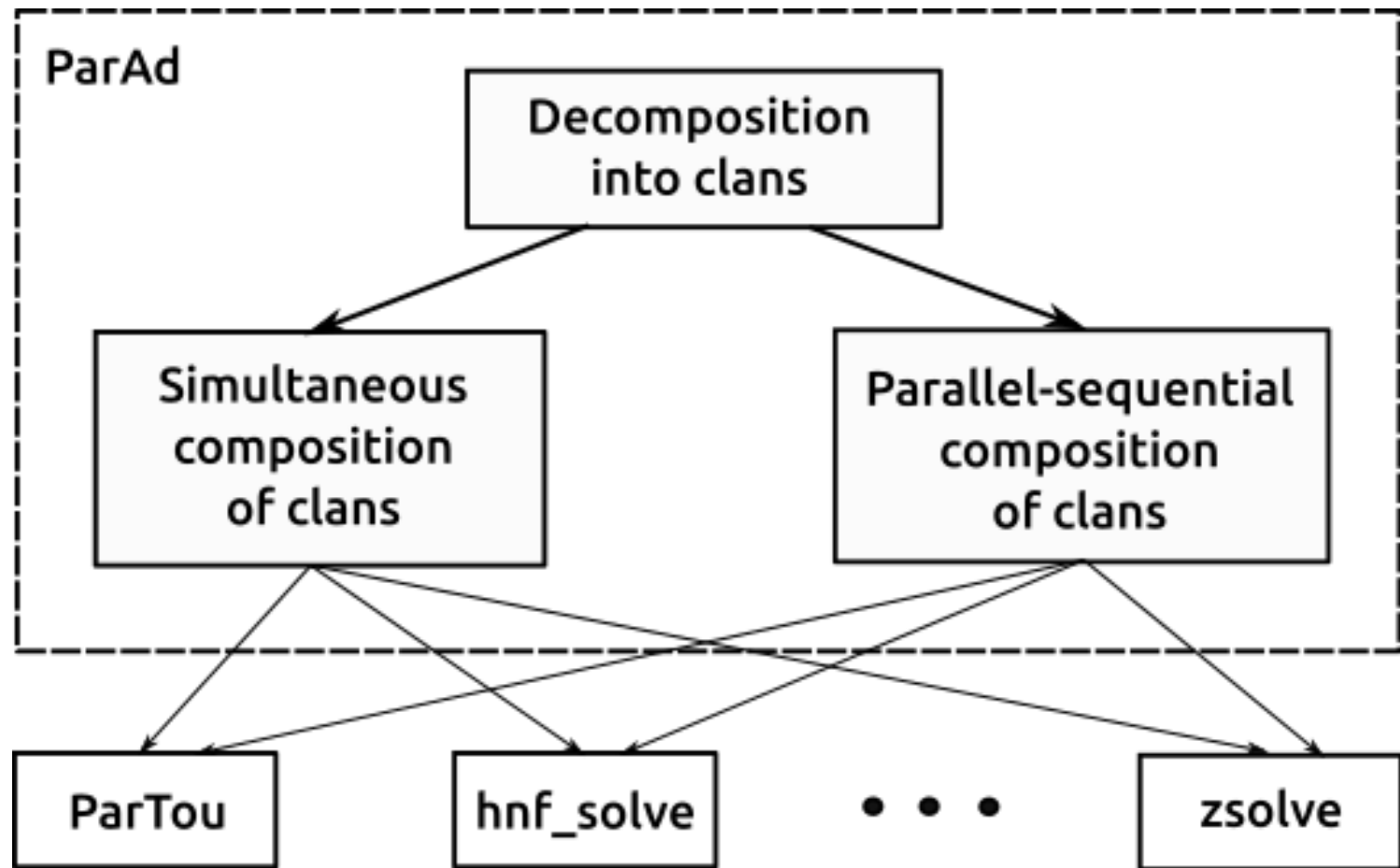
$\Downarrow 3$



$\Downarrow 6$

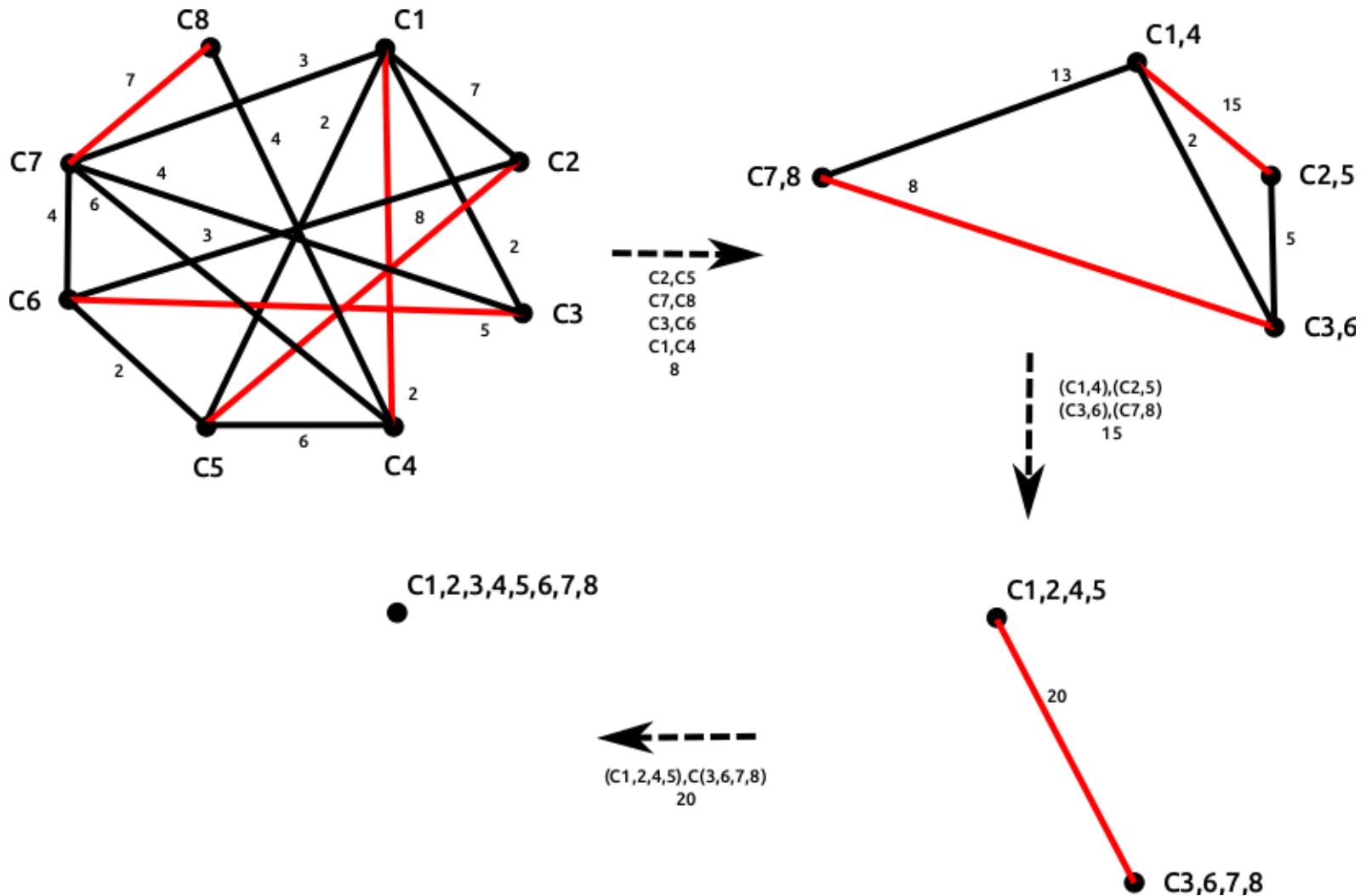


ParAd – Parallel Adriana

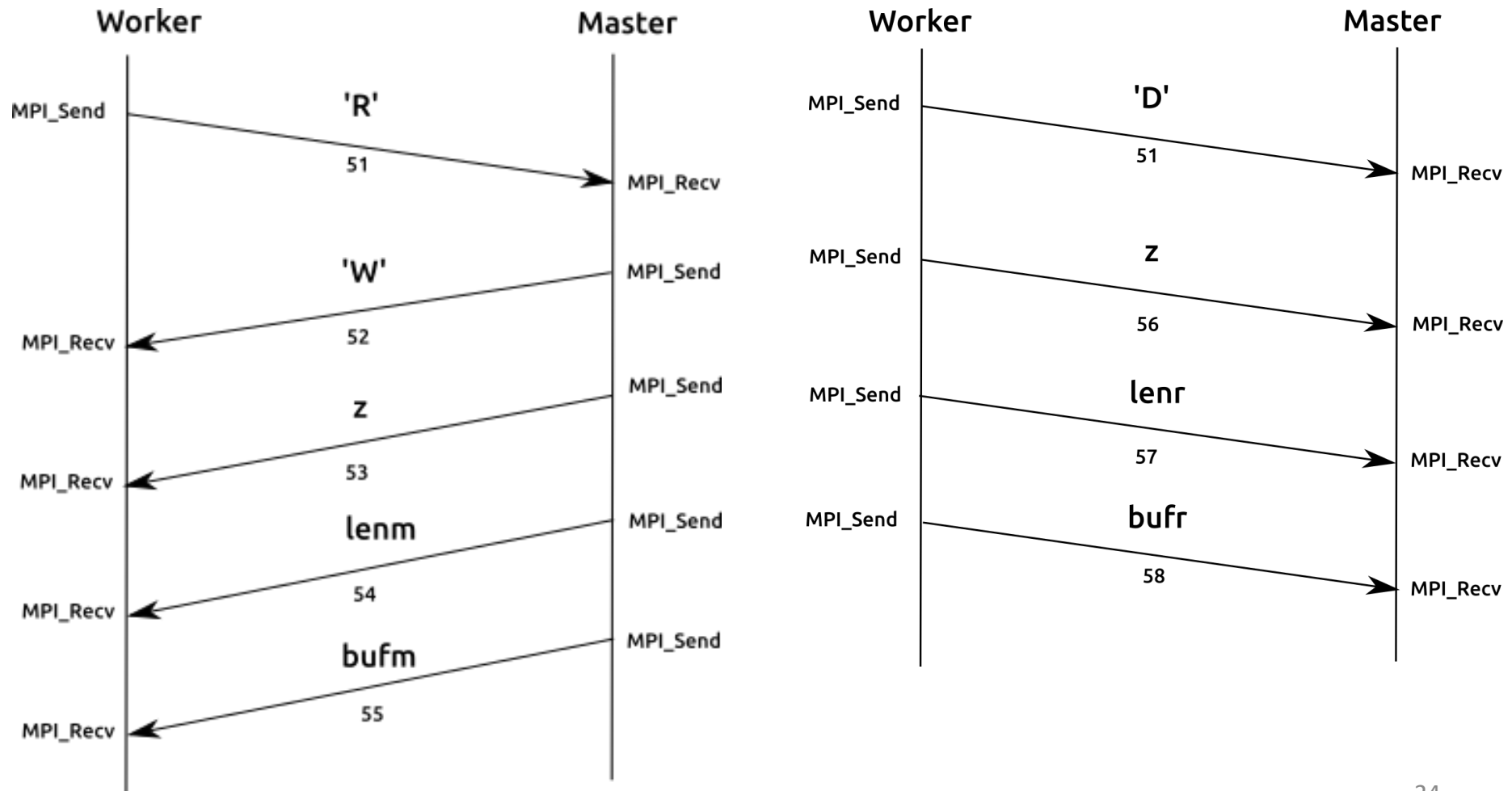


Solvers of linear systems

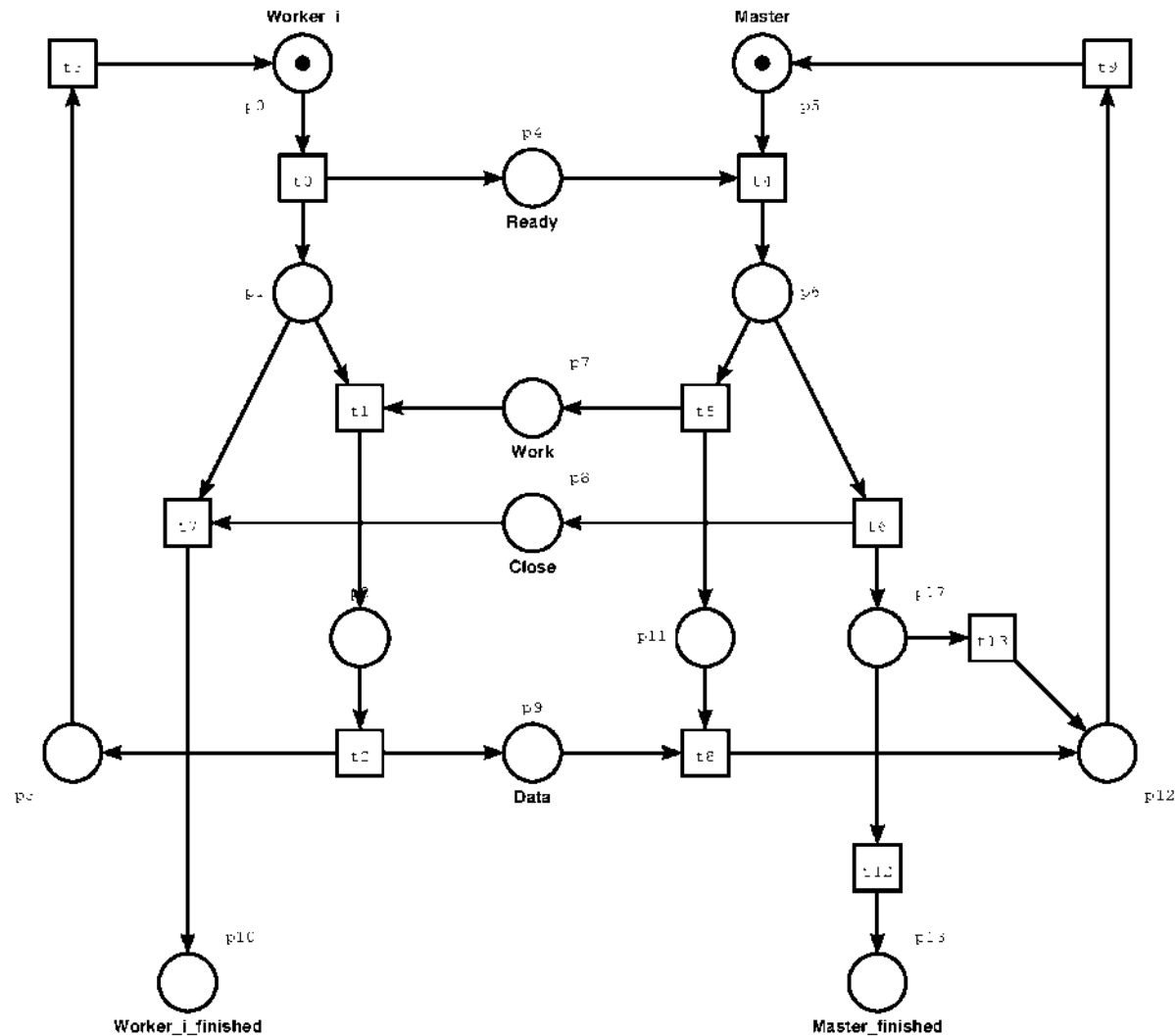
Parallel-sequential Composition of Clans



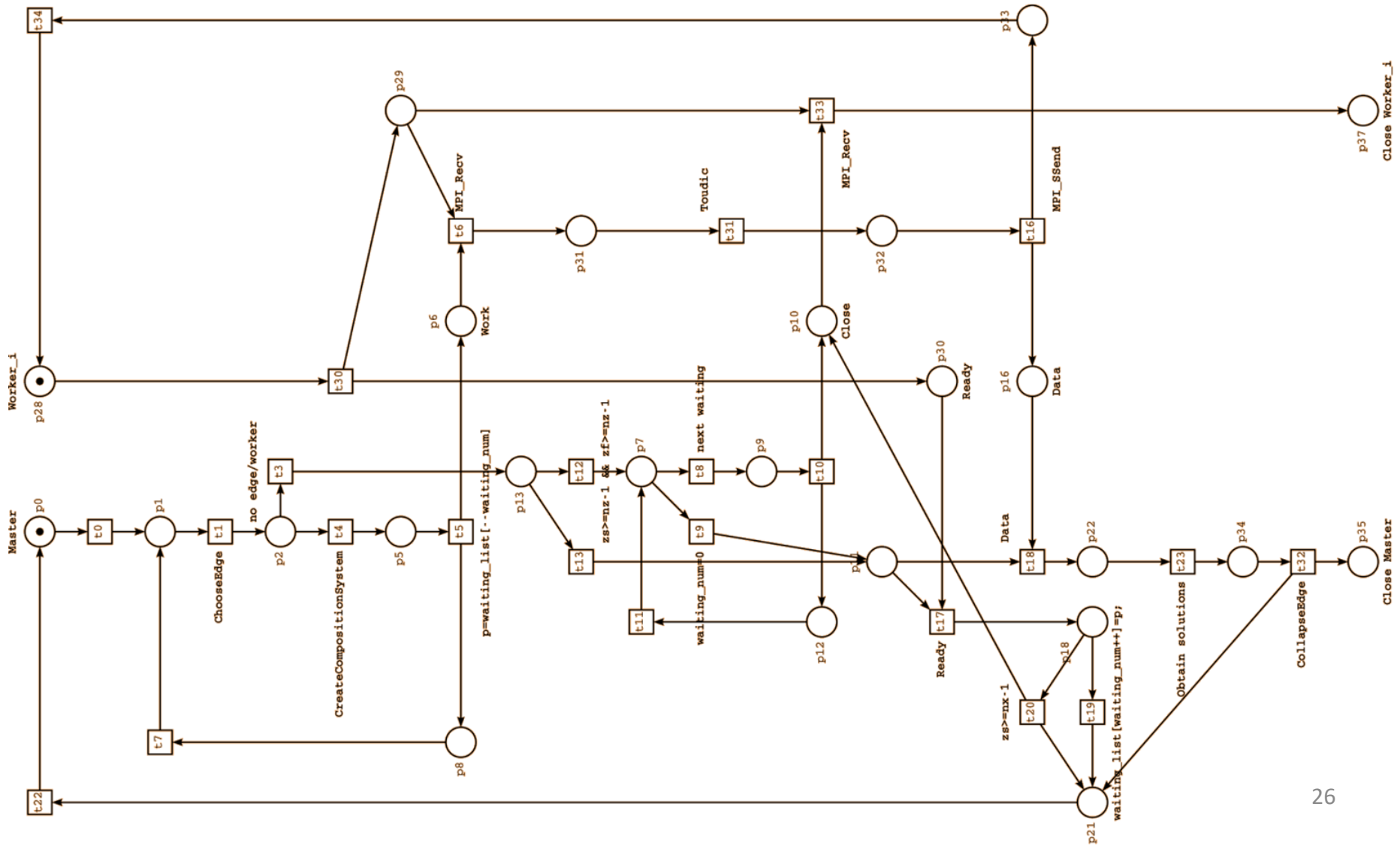
Protocols of data transmission



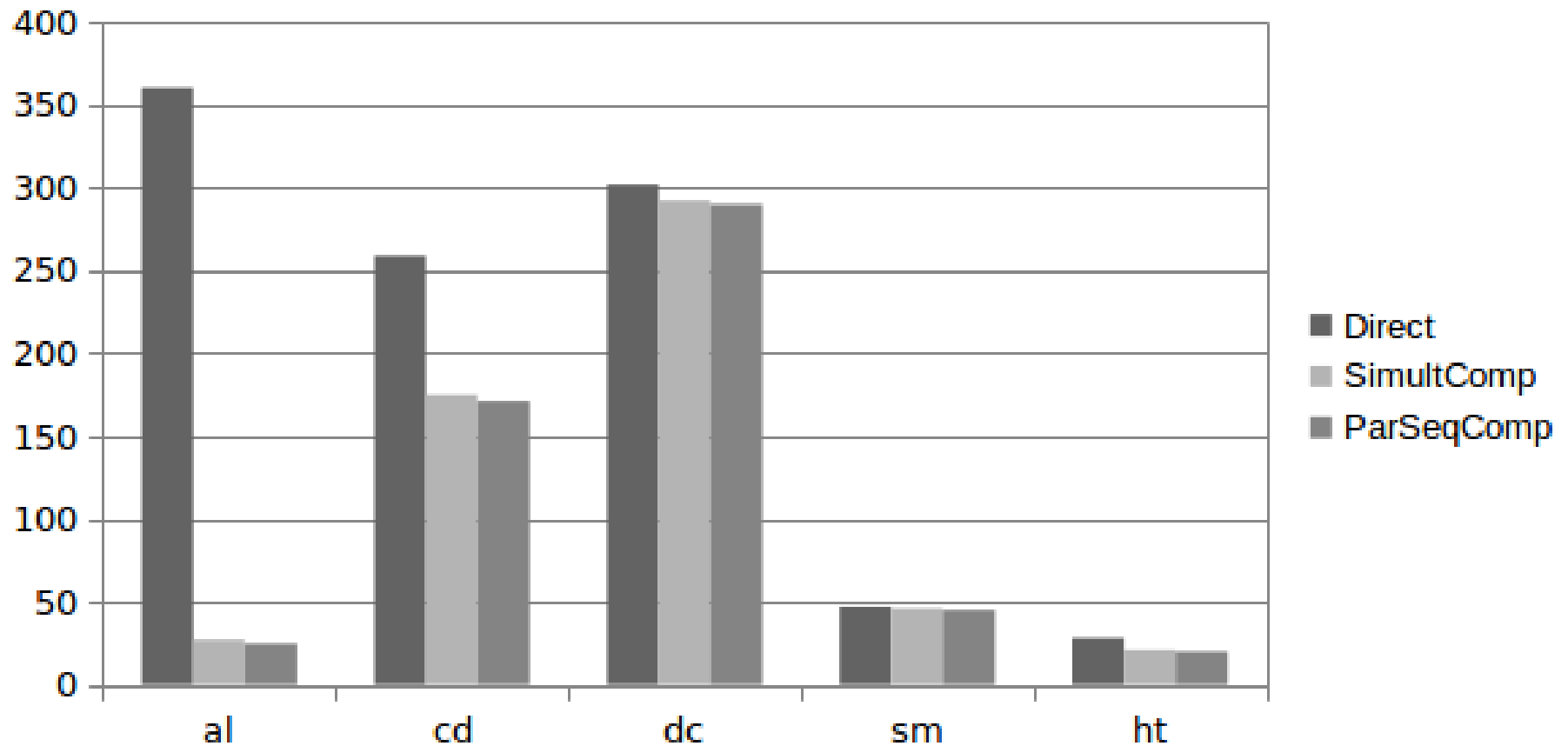
Master-Worker Basic Communication Protocol



Parallel-Sequential Composition Communication Protocol



Speed-up because of clans



Run ParAd

- **Run with mpirun**

```
>mpirun -n 5 ./ParAd -c -r zsolve tcp.spm tcp-pi.spm  
>mpirun -n 10 ./ParAd -s -t -d 1 tcp.spm tcp-ti.spm
```

- **Run with Slurm**

```
>srun -N 10 ./ParAd -s -t -d 1 tcp.spm tcp-ti.spm
```

- **SPM – simple sparse matrix format:**

```
i j a[i][j]
```

- **Check decomposability (Matrix Market Format)**

```
>toclans lp_cre_d.mtx
```

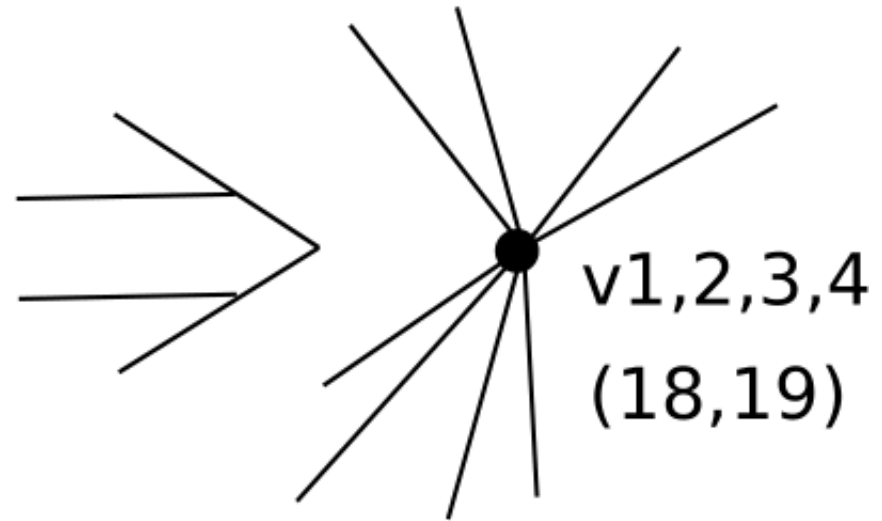
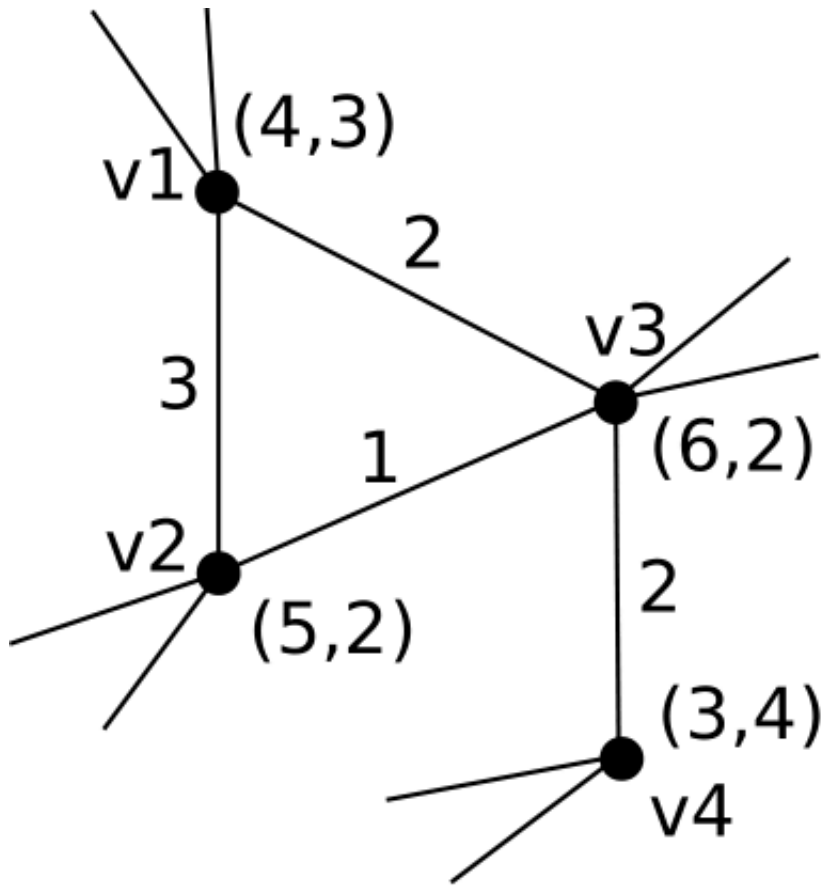
Load balancing

- ***Dynamic*** on demand – appoint a clan to a free node (version 1.1)
- ***Static*** – aggregate minimal clans into big clans according to the number of available computing nodes
- ***Hybrid*** – pre-aggregation to equal size and then dynamic scheduling clans to nodes (version 1.2)

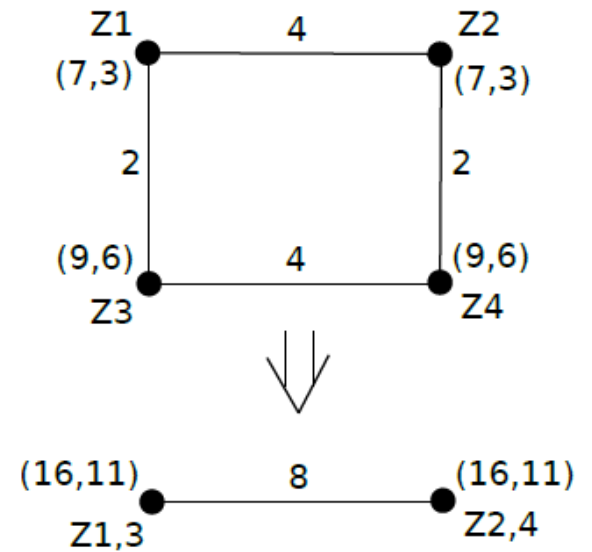
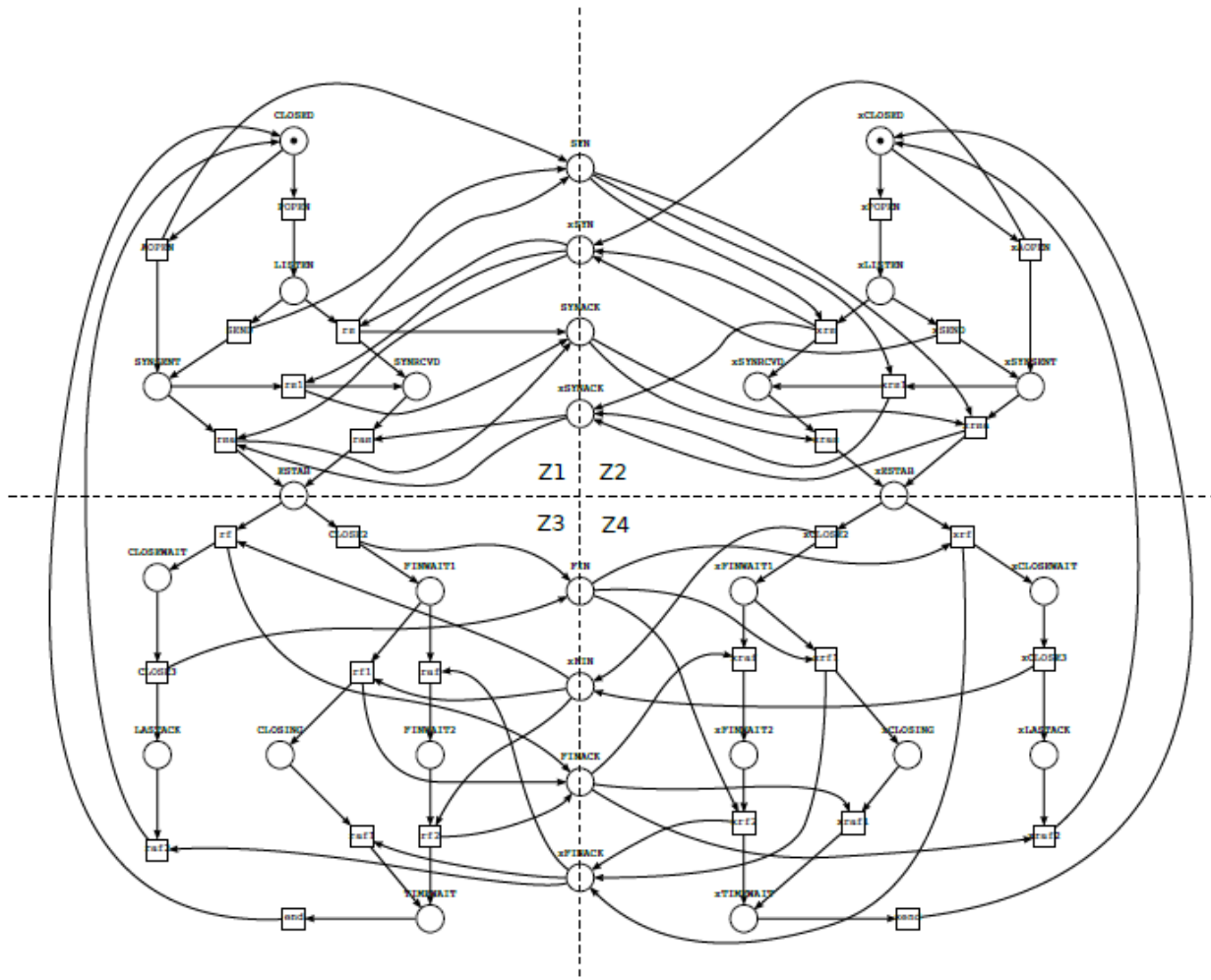
Aggregation of Clans for Workload Balancing

- **A clan is a sum of minimal clans**
- **The maximal clan size restricts granulation**
- **Many small clans lead to heavy communication load**
- **Balancing: create clans having size close to the maximal**
- **Key: -a val**
- **Aggregation speeds-up about 20%**

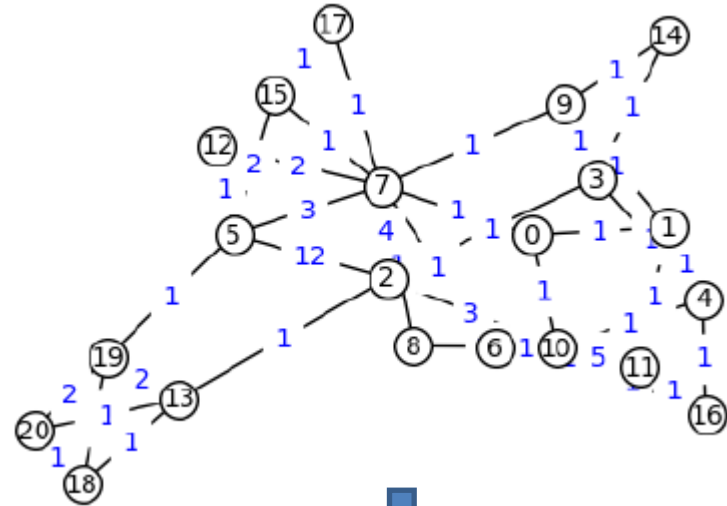
Aggregation idea



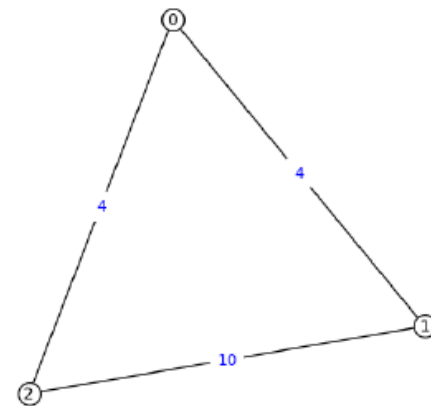
Verification of TCP



Verification of IOTP

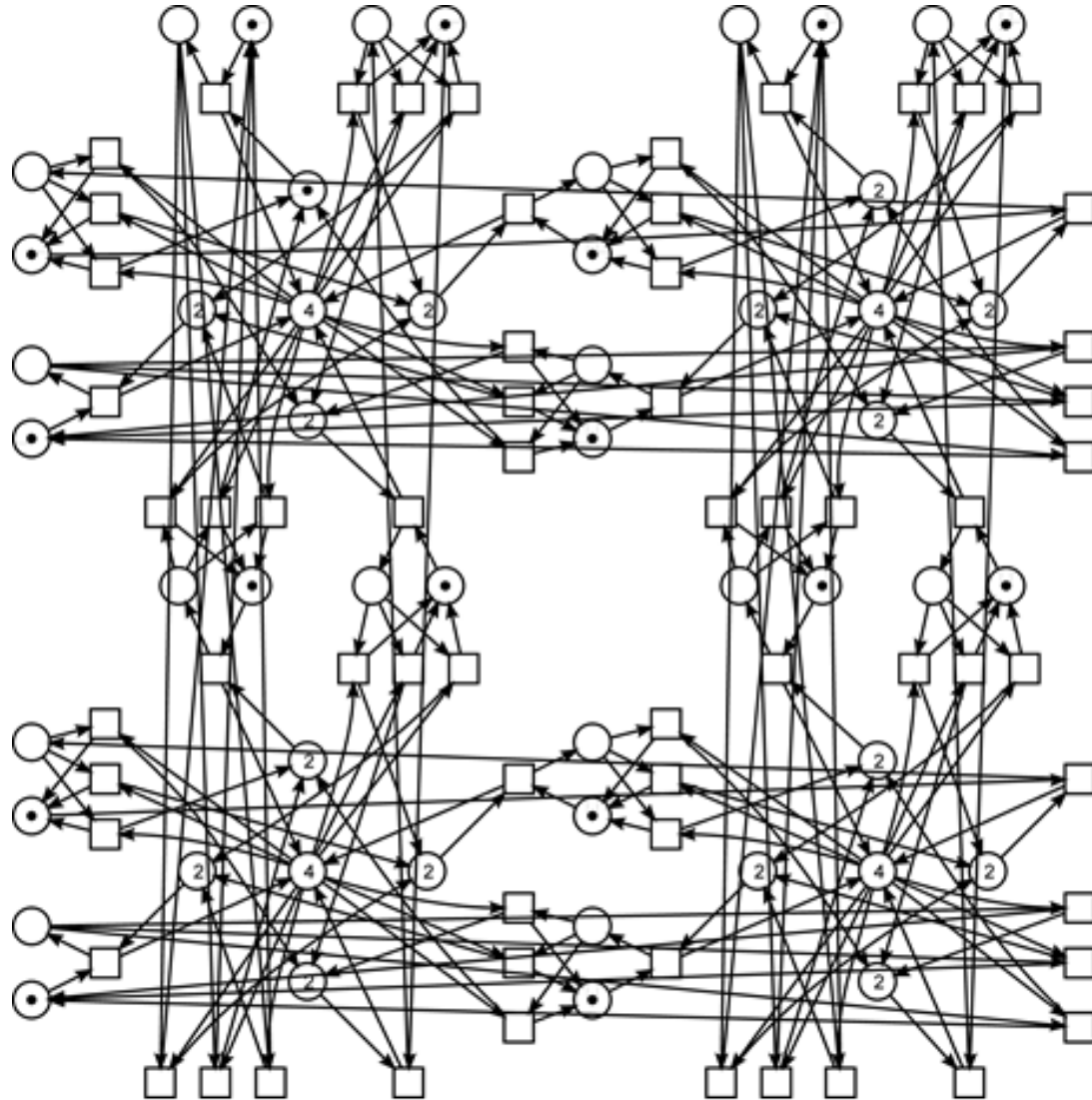


Aggregation

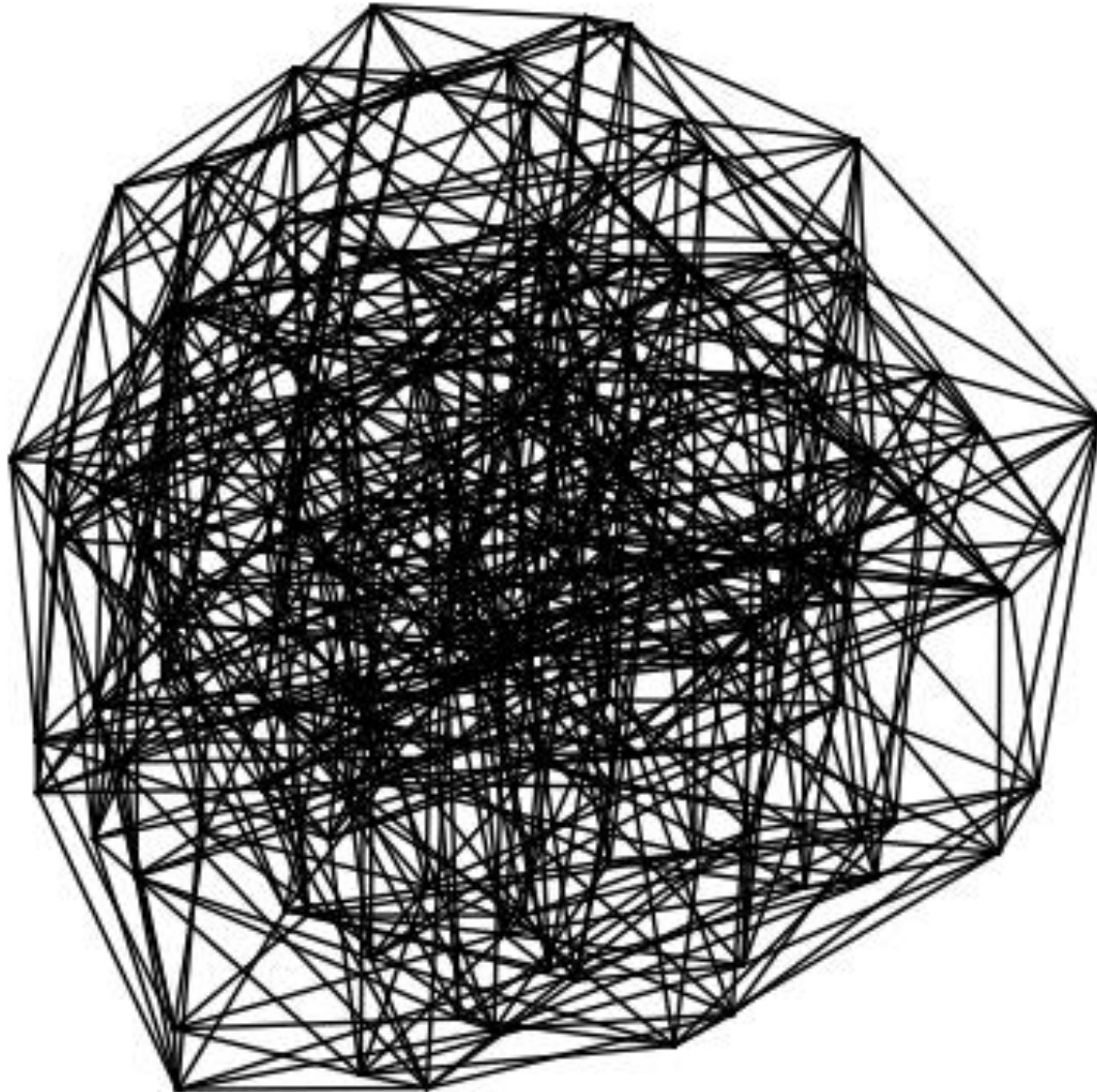


edge-cut 18, partition weight 13

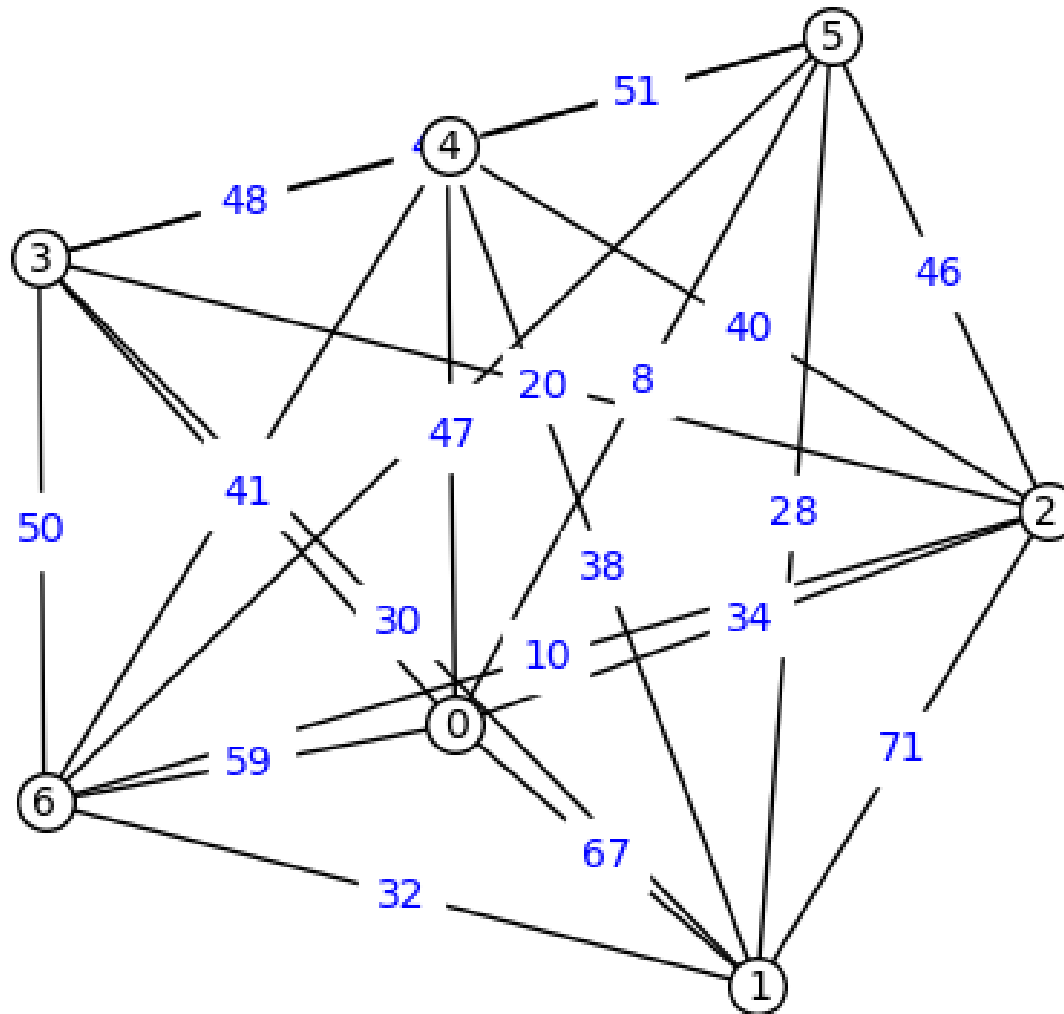
Hypertorus switching grid, 2D, 2x2 model



Decomposition of hypertorus 4D, 3x3

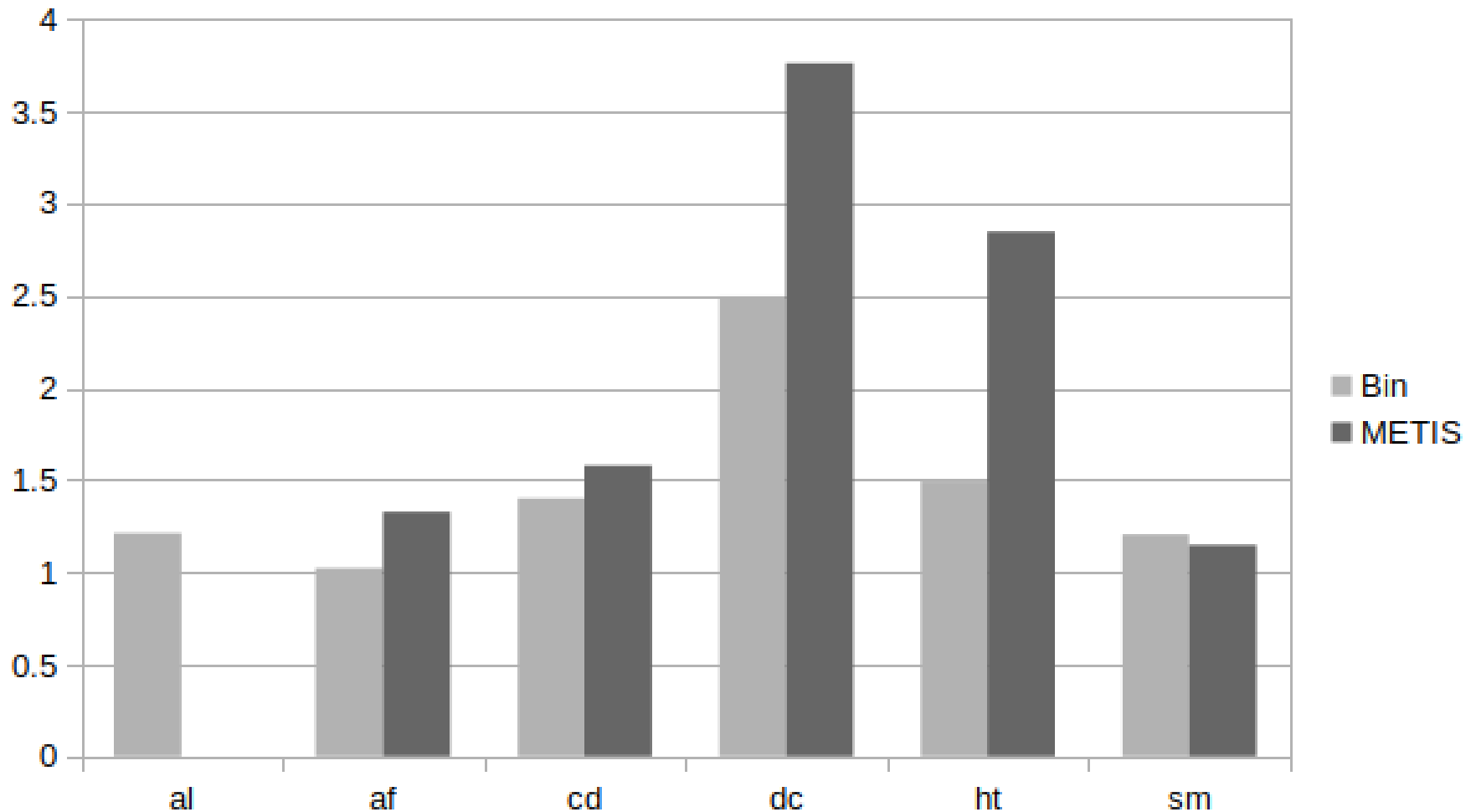


Aggregation by METIS into 7 clans



7 partitions: edge-cut 855, maximal partition weight 760

Extra speed-up because of aggregation



Conclusions

- **Additional modules for ParAd:**
 - a) **clans aggregation with METIS;**
 - b) **clans aggregation with bin packing**
- **Tested with verification of TCP, IOTP, and hypertorus grid**
- **Extra speed-up because of aggregation up to 4 times**

Recent references

- ParAd, <https://github.com/dazeorgacm/ParAd>
- Dmitry Zaitsev, Stanimire Tomov, Jack Dongarra. Solving Linear Diophantine Systems on Parallel Architectures, IEEE Transactions on Parallel and Distributed Systems, 30(5), 2019.
- Zaitsev D.A. Sequential composition of linear systems' clans, Information Sciences, Vol. 363, 2016, 292–307.
- D. A. Zaitsev, Clans of Petri Nets: Verification of protocols and performance evaluation of networks. LAP LAMBERT Academic Publishing, 2013.