Infinite Petri Nets

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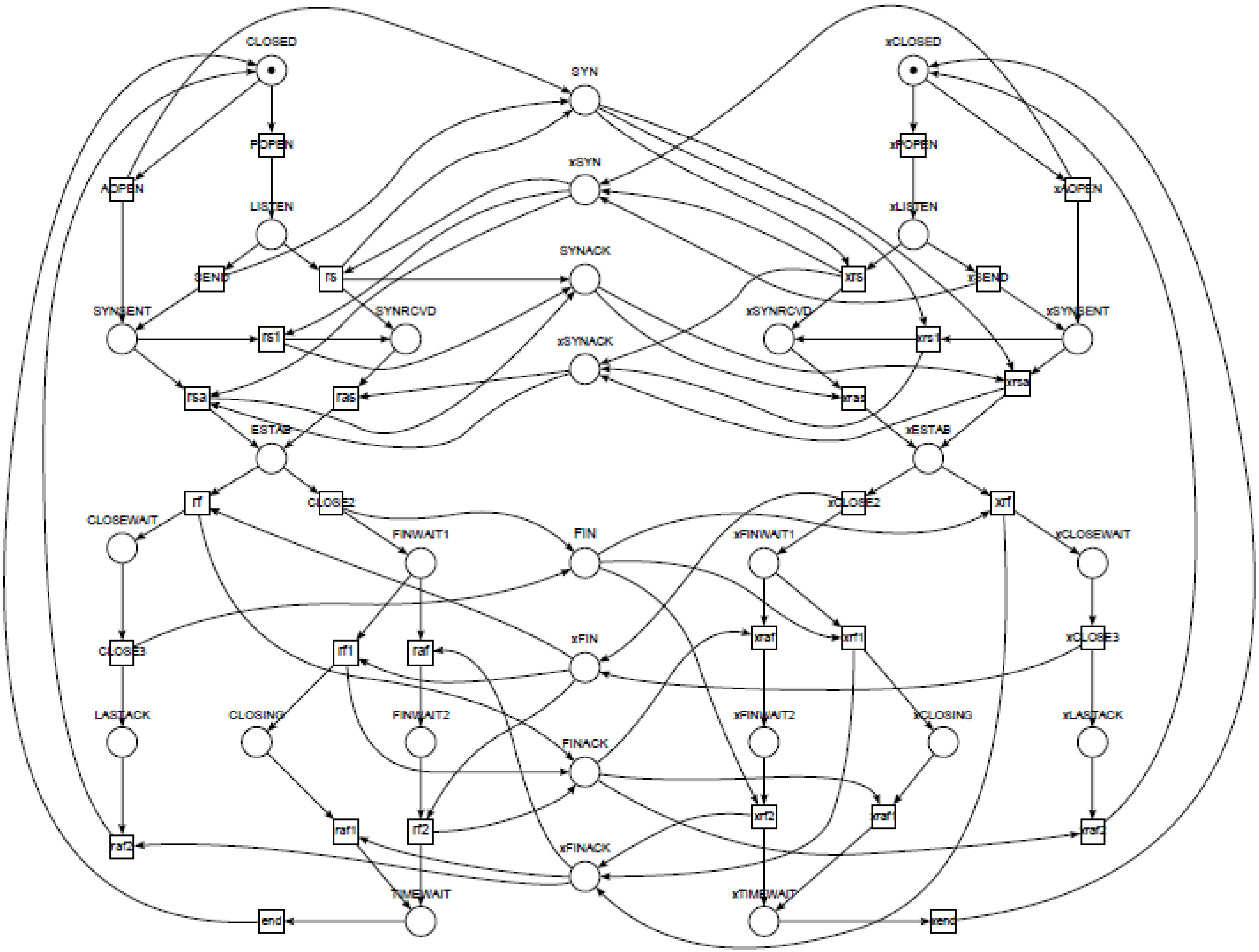
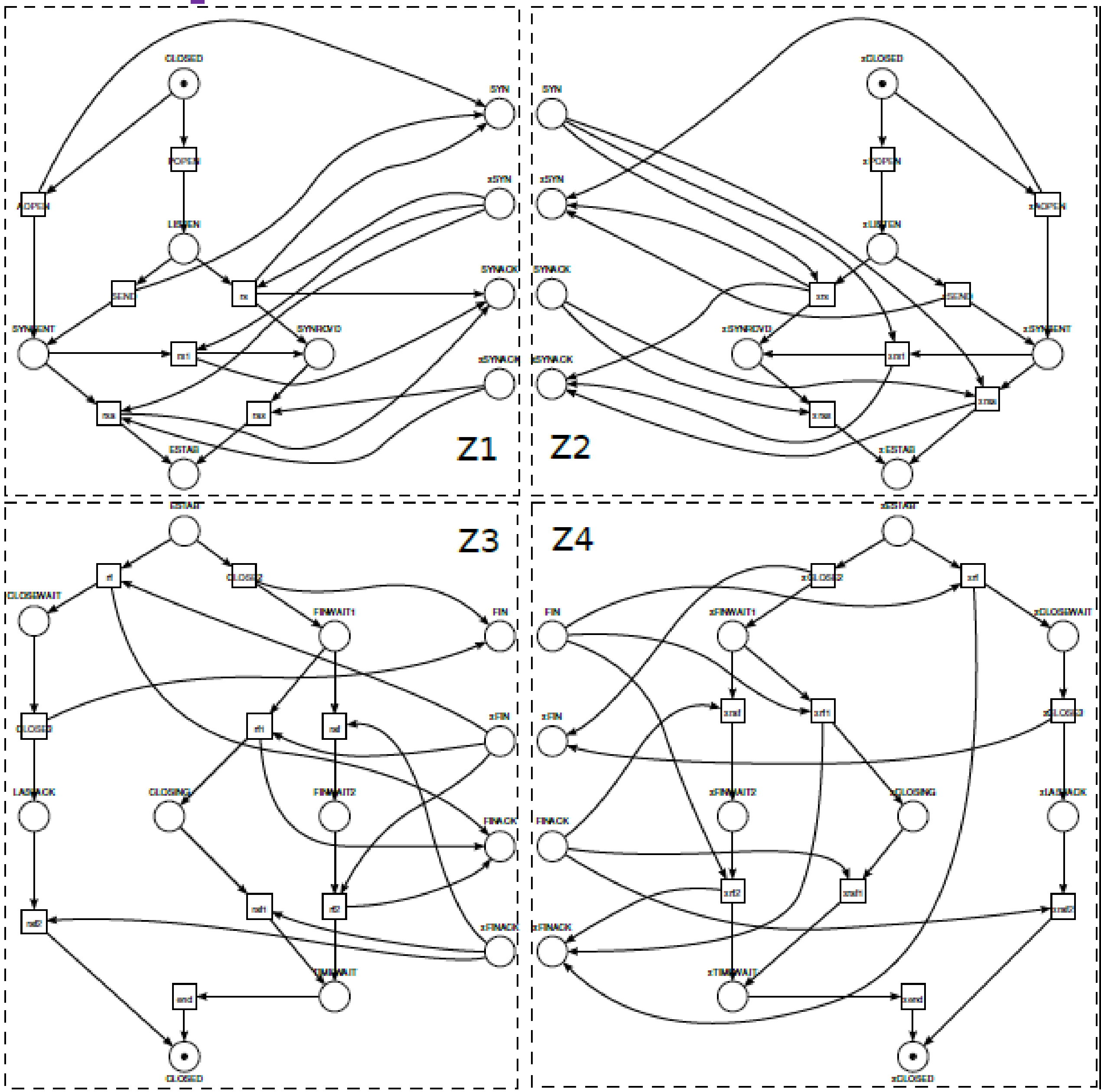
Verification of protocols

- Interaction of two systems – BGP, TCP
- Interaction of a definite number of systems – IOTP
- Building Petri net model
- LBS analysis of the model
- Compositional analysis (clans)

Model of protocol TCP



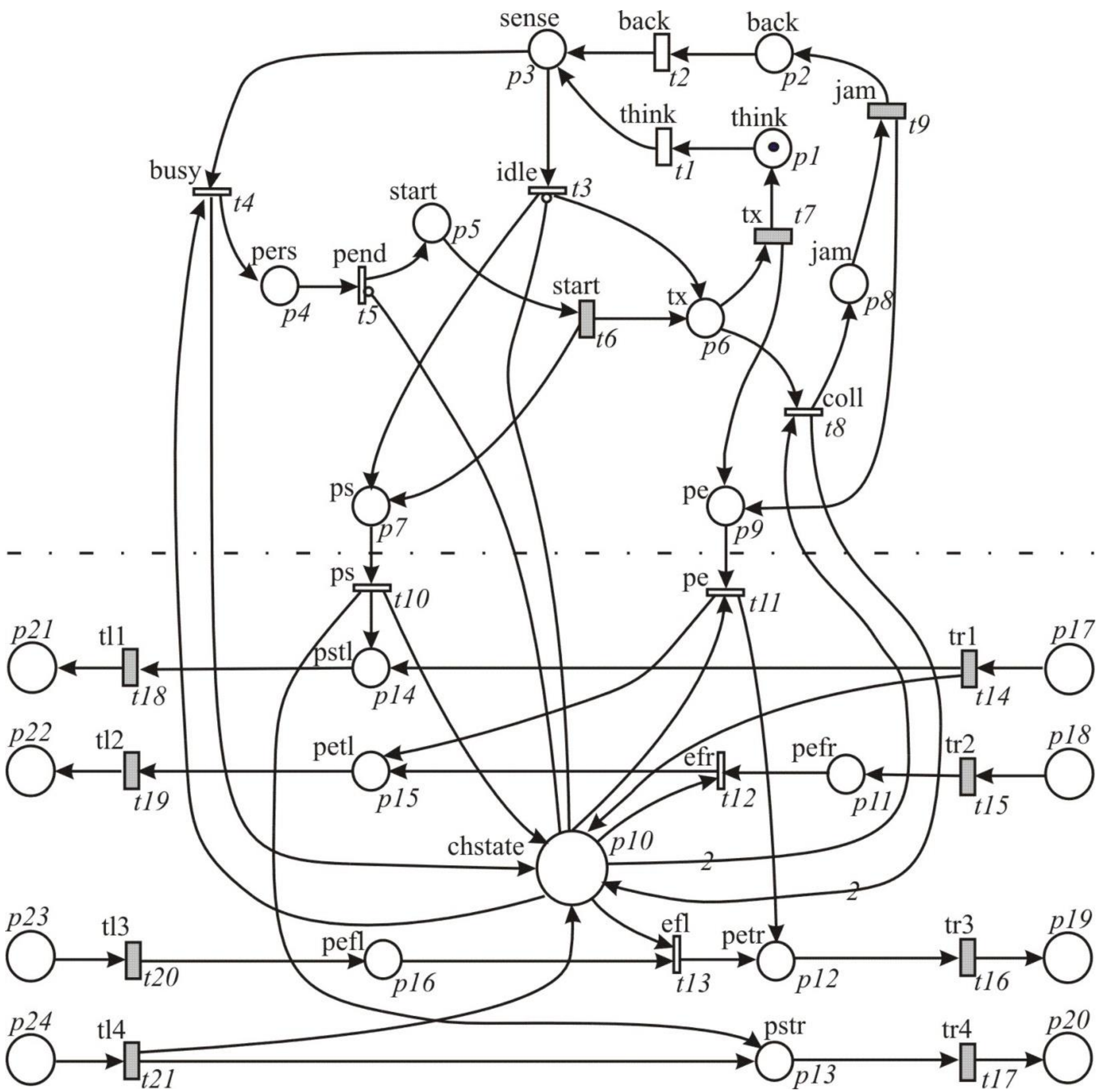
Decomposition of TCP model



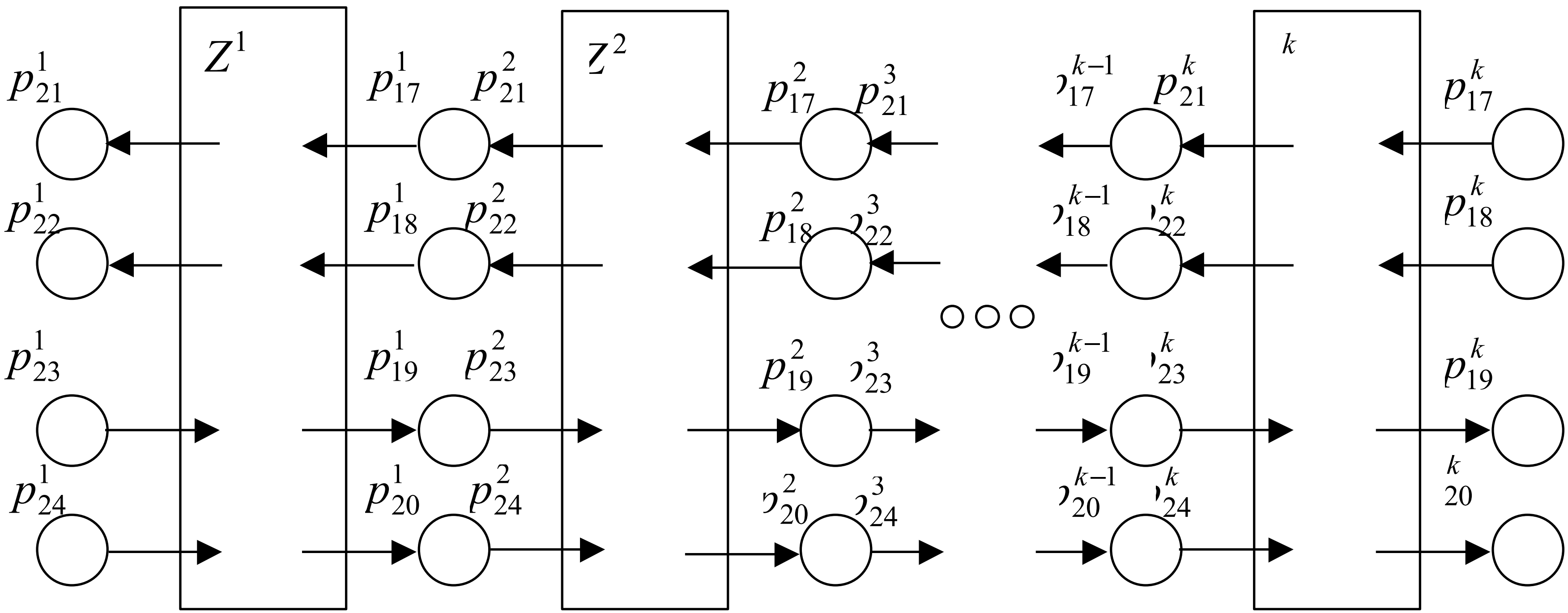
Problem of Marsan

- Verification of common bus Ethernet protocols
- Petri net model of a workstation
- How to compose the network model?
- How to analyze the network model?
- Variable and a priori unknown number of devices on the bus

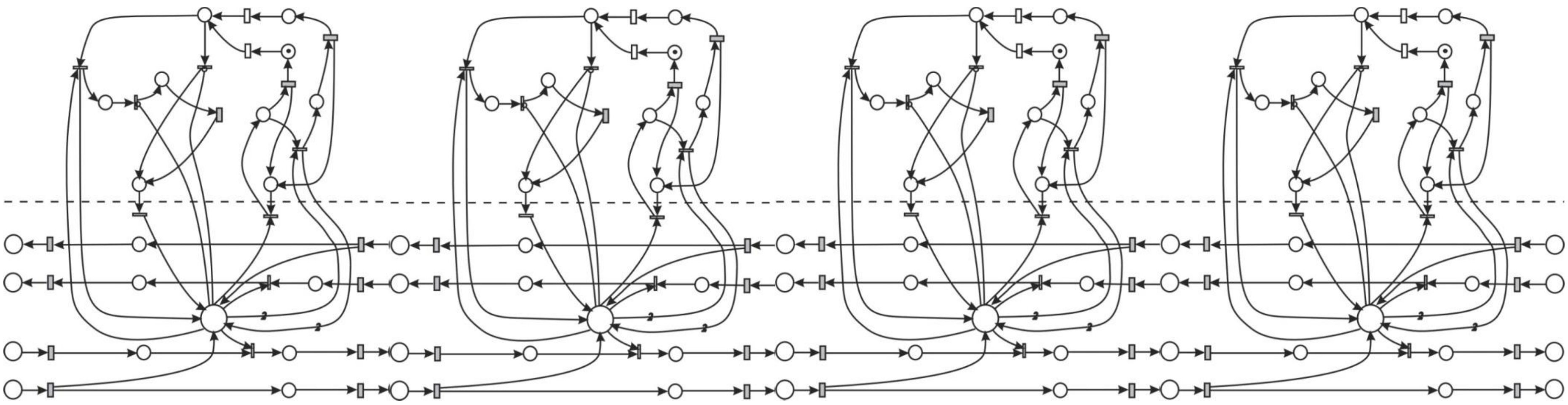
Model of a workstation



Common bus Ethernet scheme



An example of a bus with four devices



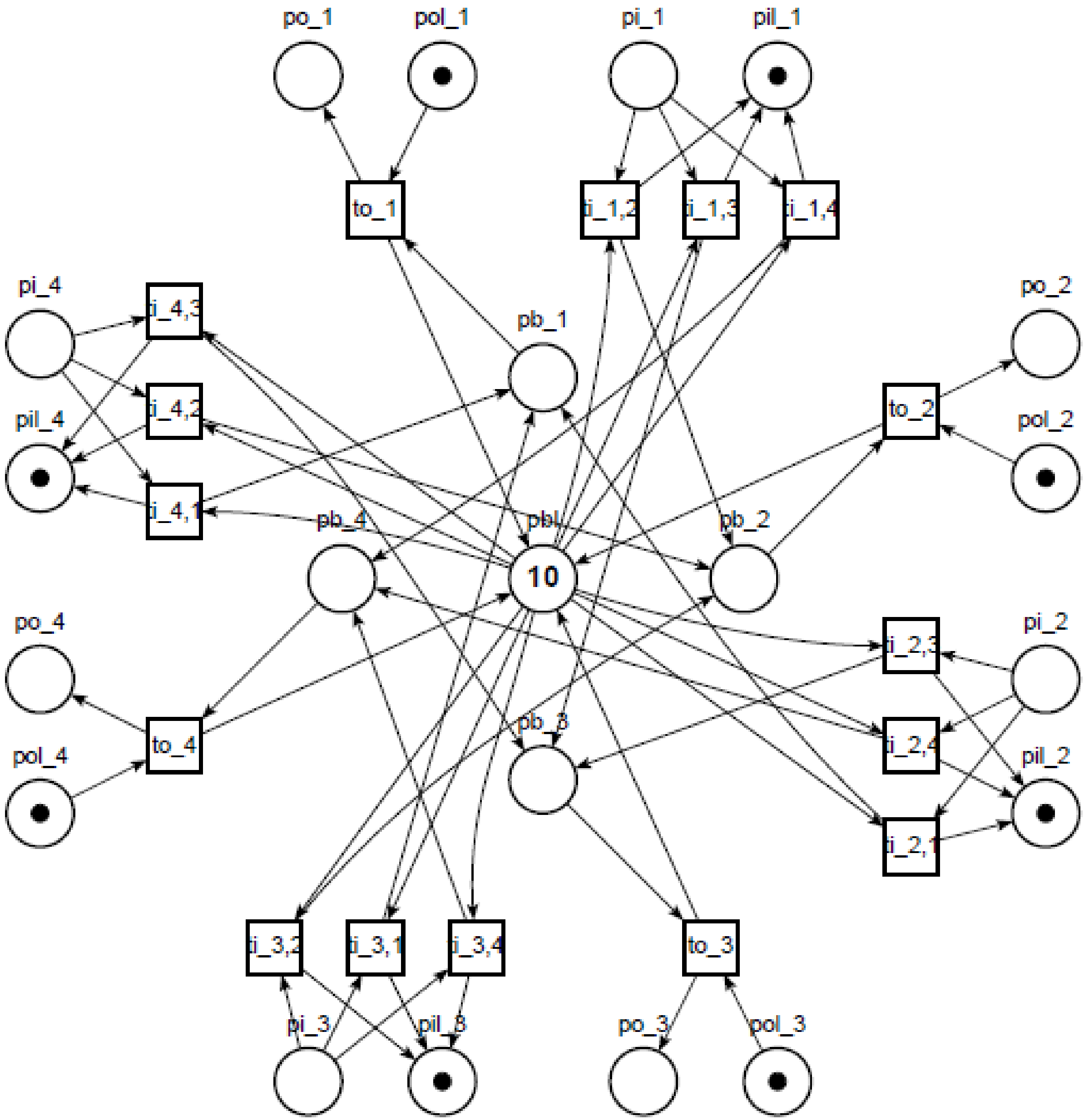
P-invariants of Ethernet model

$$
\left(
\begin{array}{l}
(p_6^i, p_8^i, p_9^i), \quad i = 1, k; \\
((p_{12}^j, p_{16}^j, p_{19}^j, p_{23}^{j=1}), \quad j = 1, k); \\
((p_{11}^j, p_{15}^j, p_{18}^j, p_{22}^{j=1}), \quad j = 1, k); \\
((p_{14}^j, p_{17}^j, p_{21}^{j=1}), \quad j = 1, k); \\
((p_{13}^j, p_{20}^j, p_{24}^{j=1}), \quad j = 1, k); \\
\left(
\begin{array}{l}
((p_{11}^j, p_{15}^j, p_{18}^j, p_{20}^j, p_{22}^{j=1}, p_{24}^{j=1}), \\
j = 1, i-1), \\
(p_1^i, p_2^i, p_3^i, p_4^i, p_5^i, p_6^i, p_7^i, p_8^i, \\
p_{10}^i, p_{12}^i, p_{15}^i, p_{17}^i, p_{19}^i, p_{22}^{i=1}, p_{24}^{i=1}), \\
((p_{12}^j, p_{14}^j, p_{16}^j, p_{17}^j, p_{19}^j, \\
p_{21}^{j=1}, p_{22}^{j=1}, p_{24}^{j=1}), j = i+1, k)
\end{array}
\right), i = 1, k
\end{array}
\right)
$$

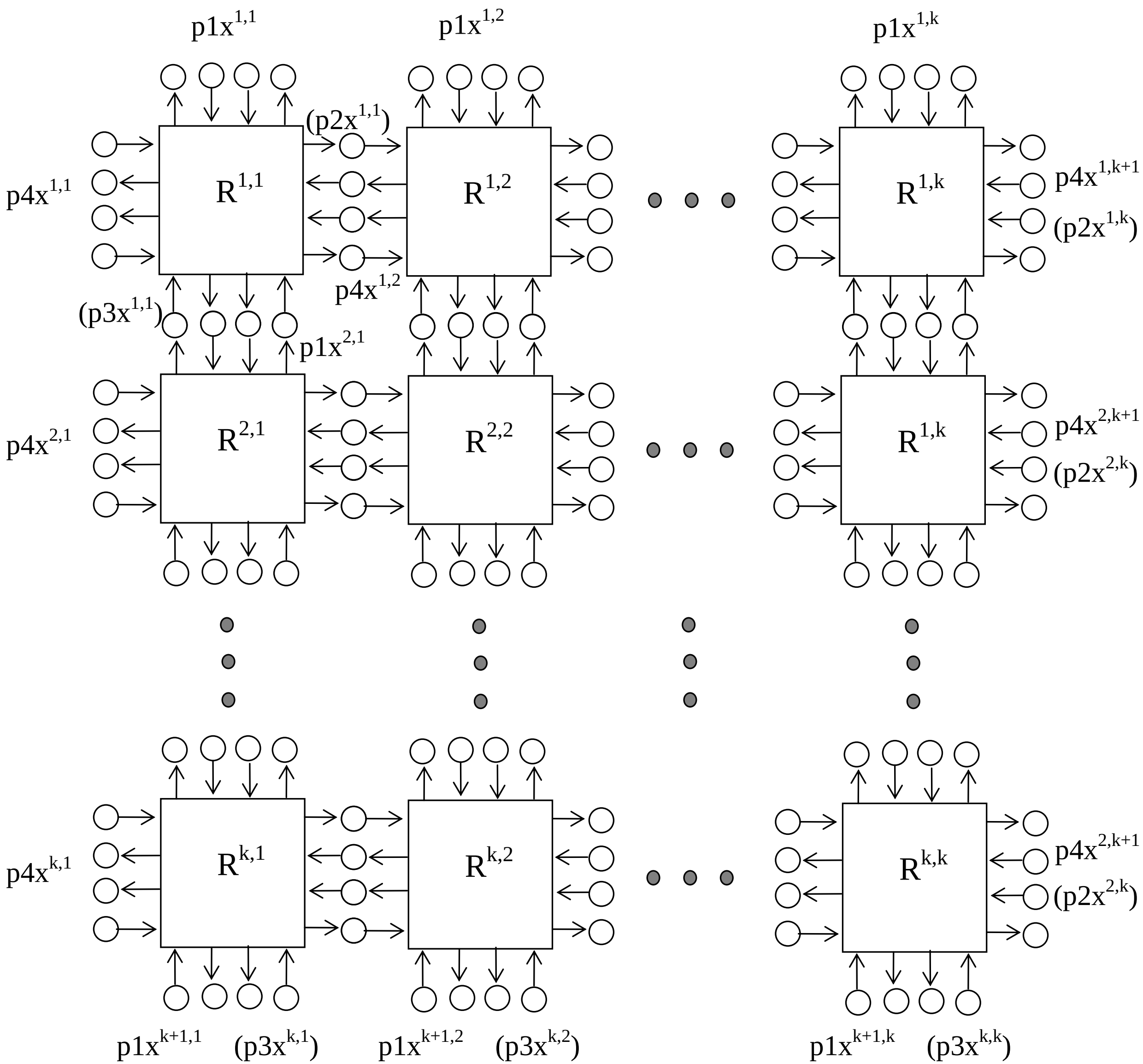
Structures of infinite nets

Structure	System	Model	Analysis
Linear	Common bus Ethernet	1987	2006
Tree-like	Switched Ethernet	2007	2007
Rectangular	Computing Grid	2008	2008
Triangular, hexagonal	Radiobroadcasting, cellular networks	2010	2010
Hypercube	Computing Grid	2008	2008
Hypertorus	Computing Grid	2012	2012

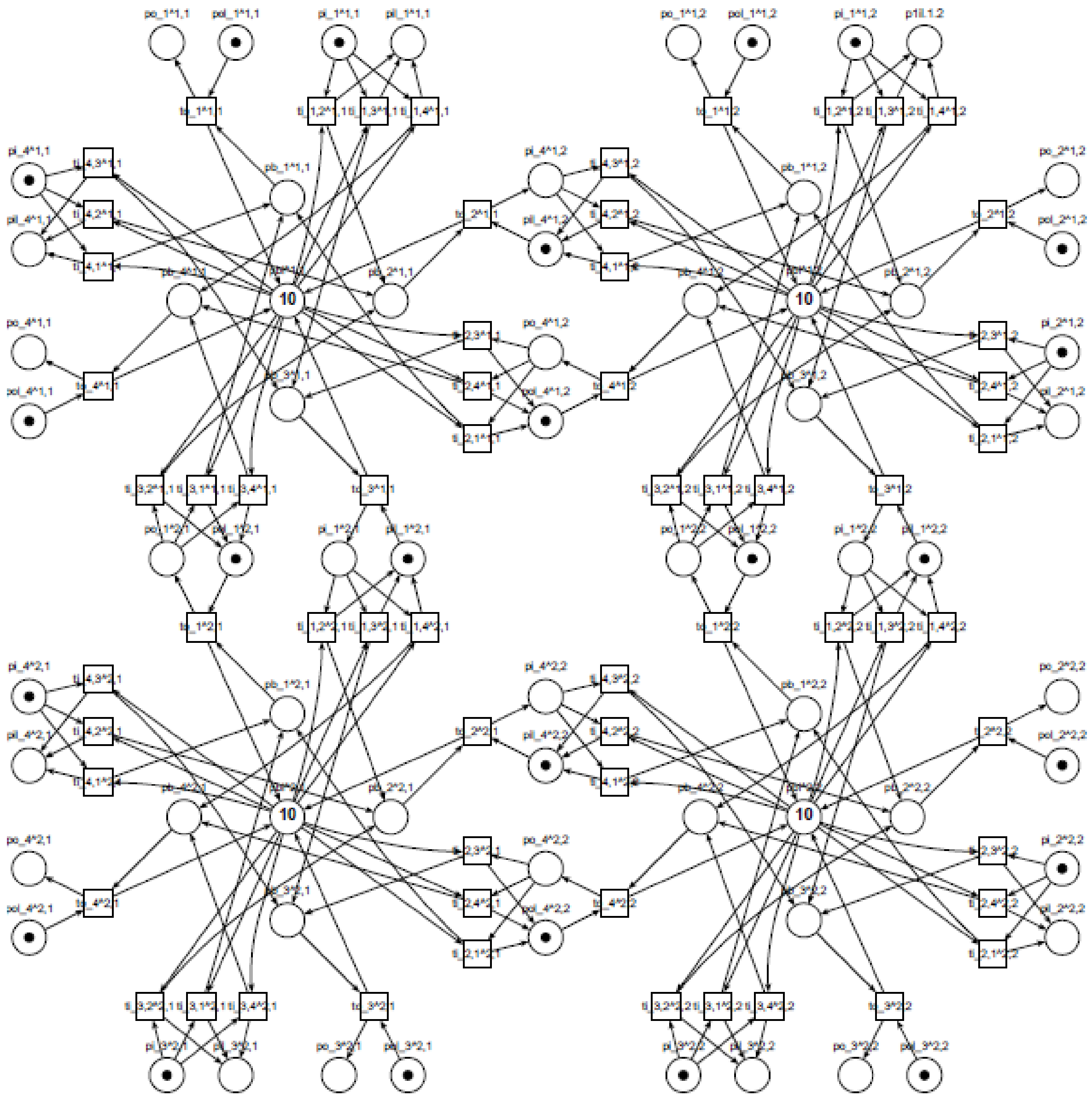
Model of rectangular grid device



Composition of rectangular grid



Open grid 2 x 2



Parametric specification of a device

$$\left(\left(\begin{pmatrix} to_u : pb_u, pol_u \to po_u, pbl \end{pmatrix} \atop \begin{pmatrix} ti_{u,v} : pi_u, pbl \to pb_v, pil_u \end{pmatrix}, \quad v = \overline{1, np}, \quad v \neq u \right), u = \overline{1, np}\right)$$

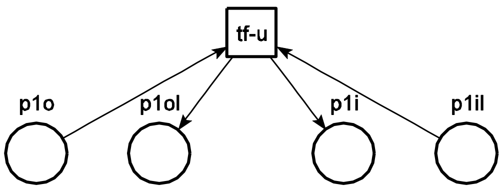
Parametric specification of open grid

$$\left(\left(\begin{array}{l}\left(to_1^{i,j} : pol_1^{i,j}, pb_1^{i,j} -> po_1^{i,j}, pbl^{i,j}\right), \\ \left(ti_{1,v}^{i,j} : pi_1^{i,j}, pbl^{i,j} -> pil_1^{i,j}, pb_v^{i,j}\right), v = 2,3,4, \\ \left(to_3^{i,j} : pil_1^{i+1,j}, pb_3^{i,j} -> pi_1^{i+1,j}, pbl^{i,j}\right), \\ \left(ti_{3,v}^{i,j} : po_1^{i+1,j}, pbl^{i,j} -> pol_1^{i+1,j}, pb_v^{i,j}\right), \quad v = 1,2,4, \\ \left(to_4^{i,j} : pol_4^{i,j}, pb_4^{i,j} -> po_4^{i,j}, pbl^{i,j}\right), \\ \left(ti_{4,v}^{i,j} : pi_4^{i,j}, pbl^{i,j} -> pil_4^{i,j}, pb_v^{i,j}\right), v = 1,2,3, \\ \left(to_2^{i,j} : pil_4^{i,j+1}, xb_2^{i,j} -> pi_4^{i,j+1}, pbl^{i,j}\right), \\ \left(ti_{2,v}^{i,j} : po_4^{i,j+1}, pbl^{i,j} -> pol_4^{i,j+1}, pb_v^{i,j}\right), \quad v = 1,3,4\end{array}\right), i = \overline{1,k}, j = \overline{1,k}\right)$$

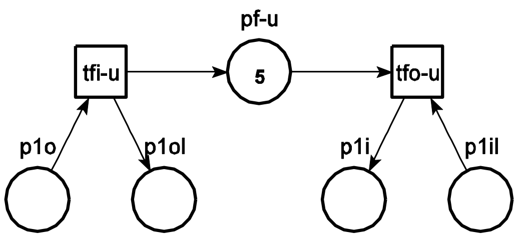
Edge conditions

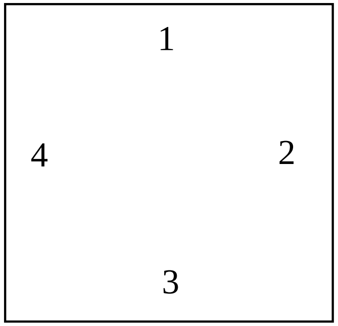
- Open grid
- Terminal (customer) device
- Connection of (opposite) edges
- Truncated communication device

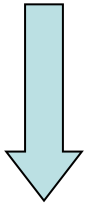
Edges



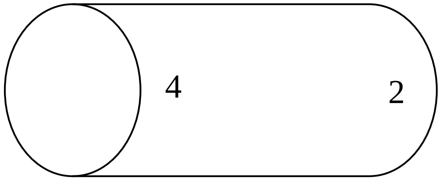
Terminal
device

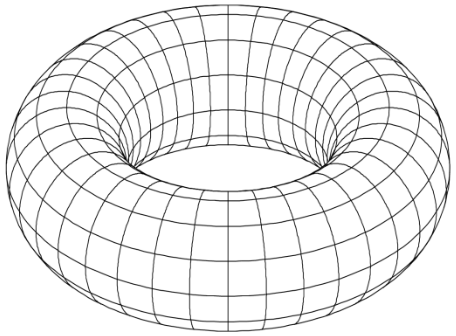


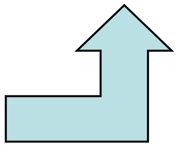




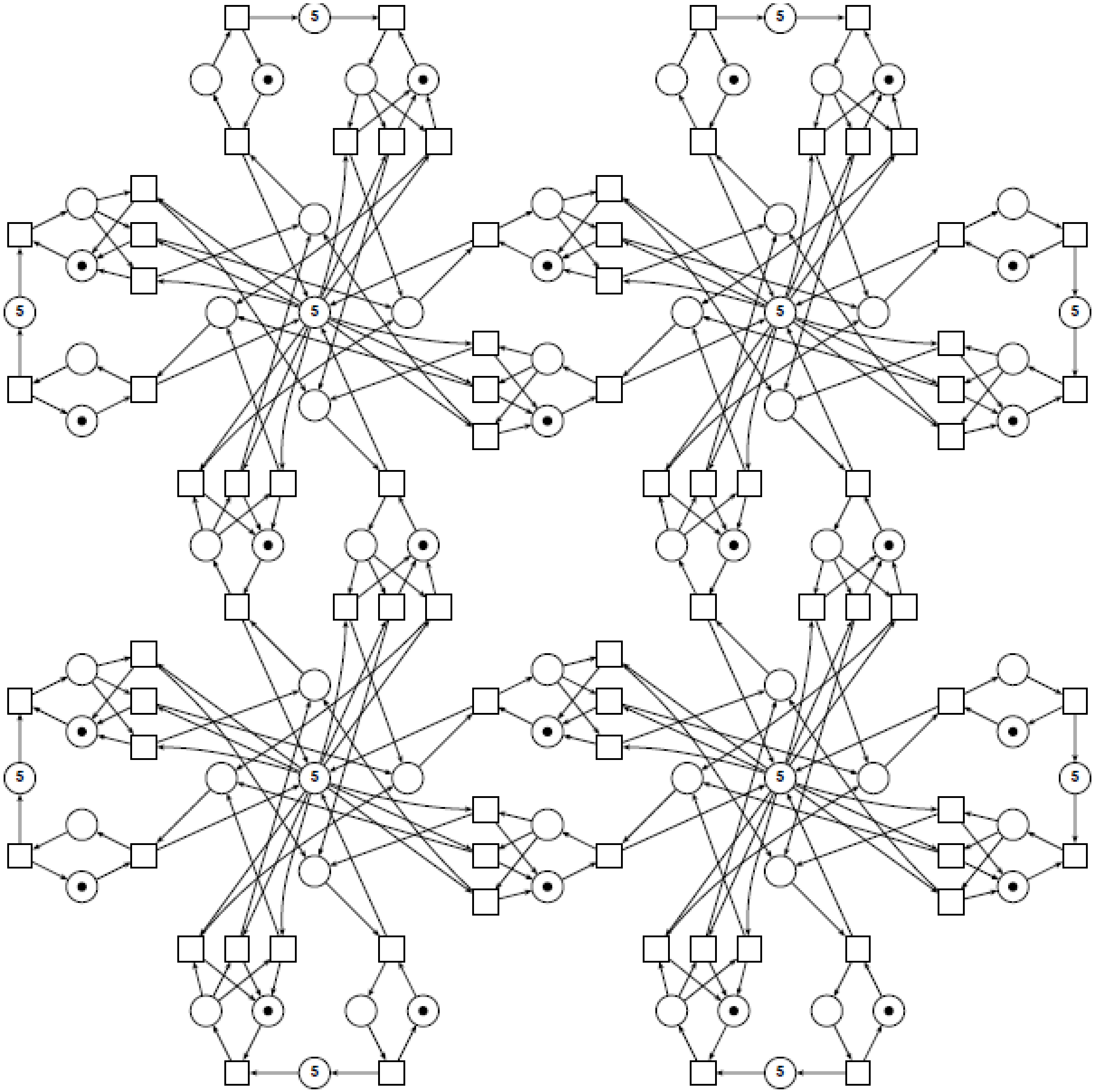
Connection
of edges



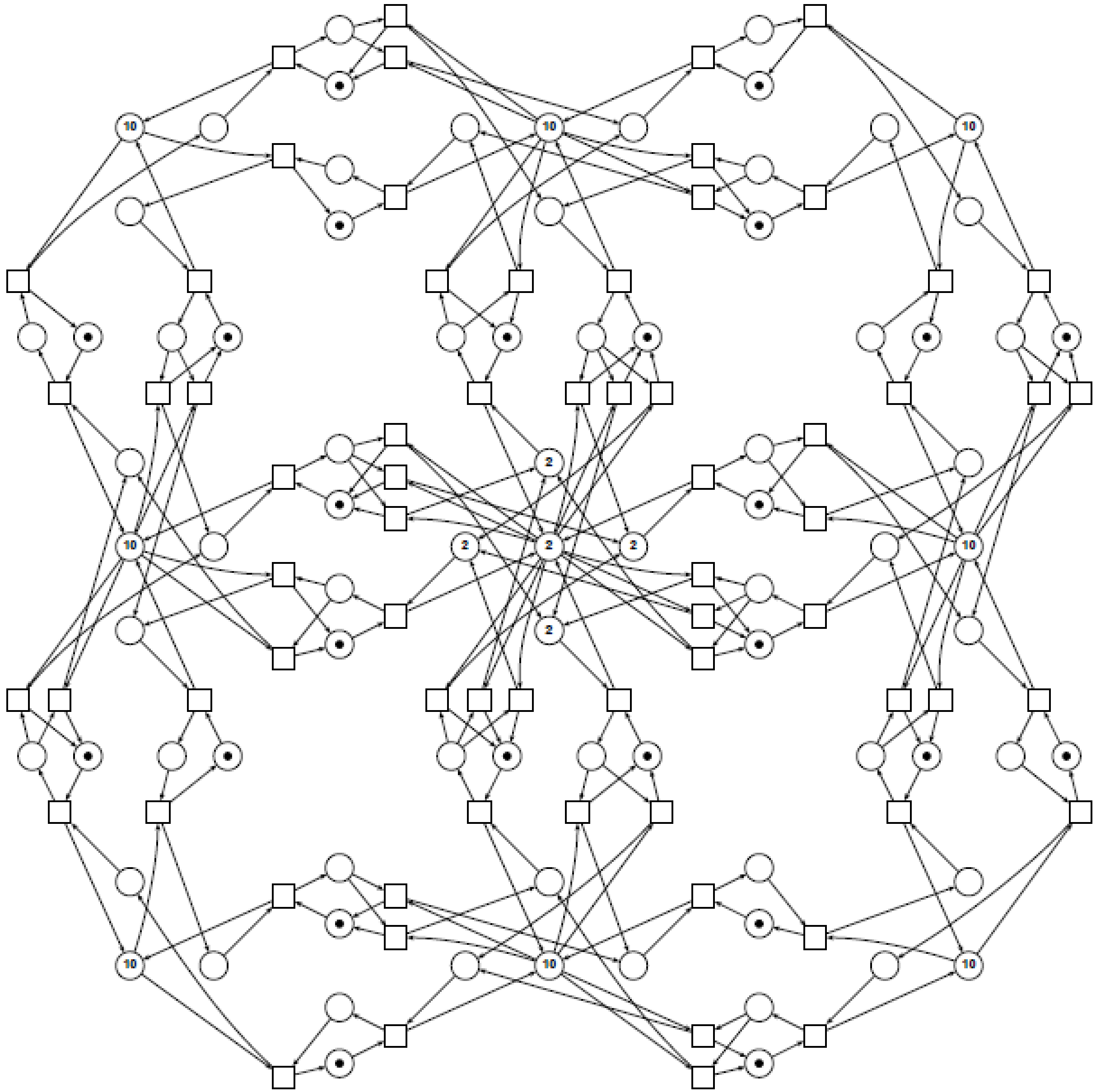




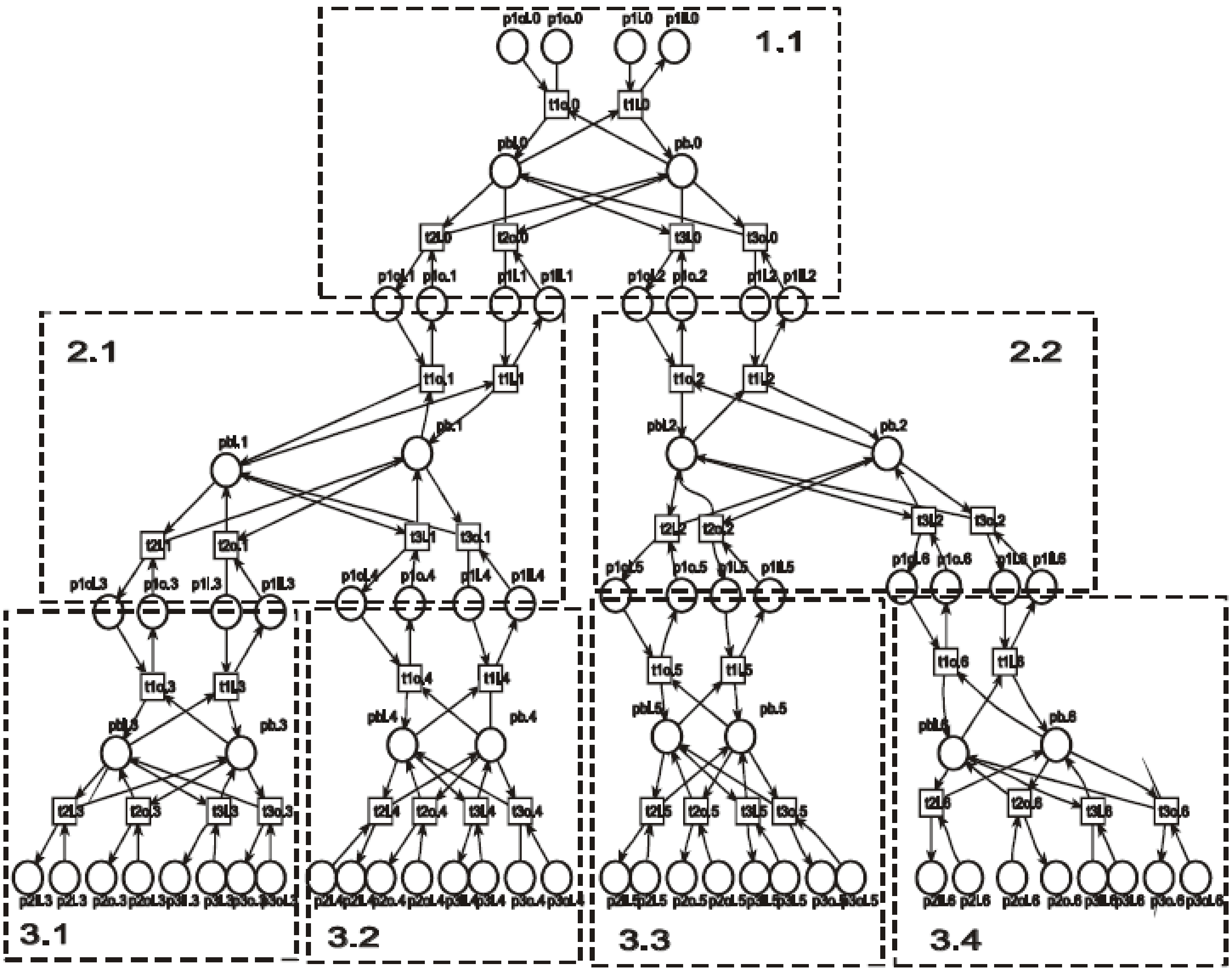
Closed grid 2 x 2



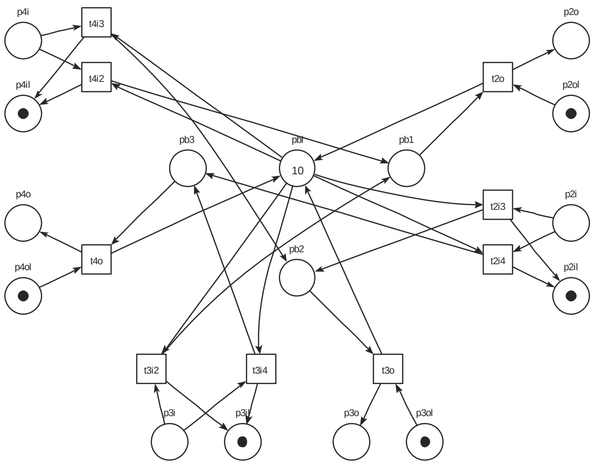
Truncated device on edges

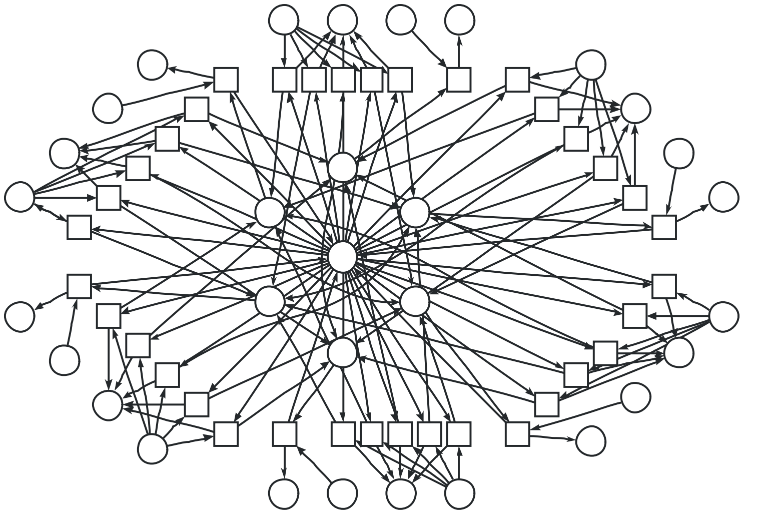


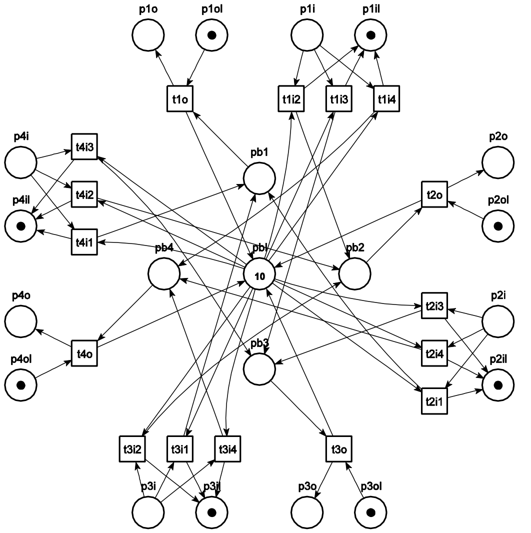
Tree-like structures



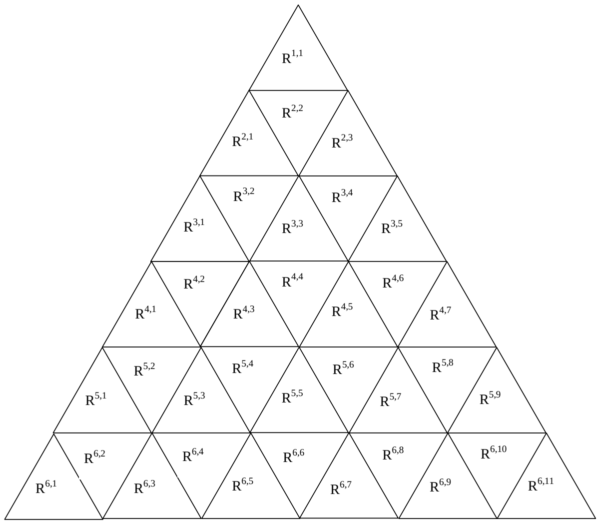
Devices: triangular, rectangular and hexagonal



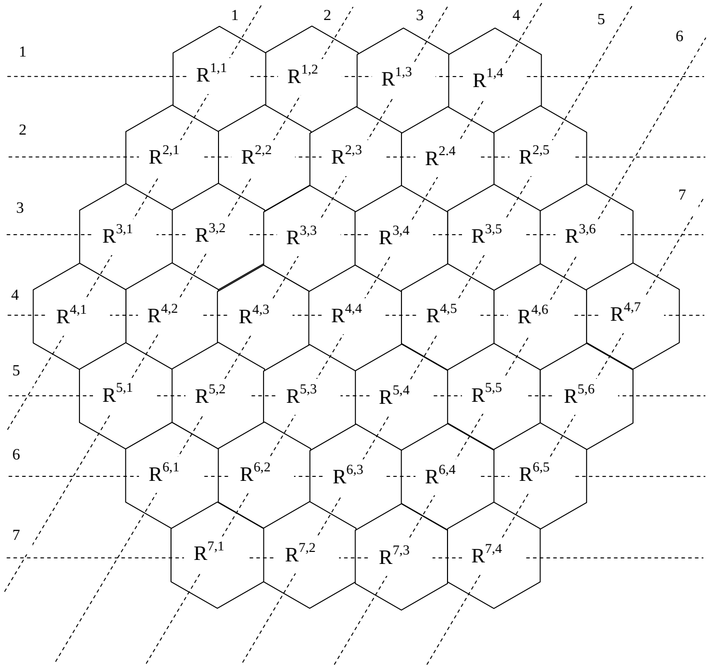




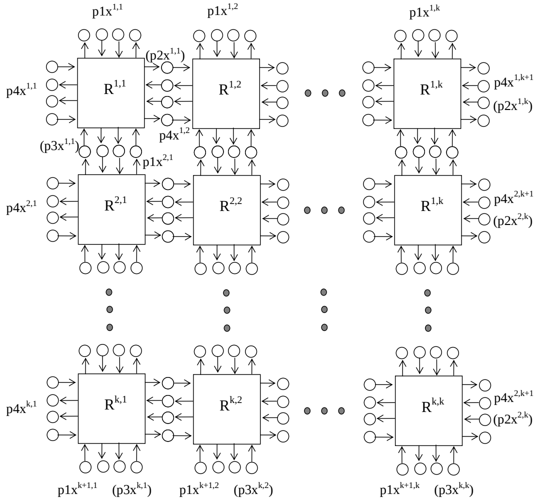
Grids



R^{1,1}
R^{2,1} R^{2,2} R^{2,3}
R^{3,1} R^{3,2} R^{3,3} R^{3,4} R^{3,5}
R^{4,1} R^{4,2} R^{4,3} R^{4,4} R^{4,5} R^{4,6} R^{4,7}
R^{5,1} R^{5,2} R^{5,3} R^{5,4} R^{5,5} R^{5,6} R^{5,7} R^{5,8} R^{5,9}
R^{6,1} R^{6,2} R^{6,3} R^{6,4} R^{6,5} R^{6,6} R^{6,7} R^{6,8} R^{6,9} R^{6,10} R^{6,11}



1 2 3 4 5 6 7
R^{1,1} R^{1,2} R^{1,3} R^{1,4}
R^{2,1} R^{2,2} R^{2,3} R^{2.4} R^{2,5}
R^{3,1} R^{3,2} R^{3,3} R^{3,4} R^{3,5} R^{3,6}
R^{4,1} R^{4,2} R^{4,3} R^{4,4} R^{4,5} R^{4,6} R^{4,7}
R^{5,1} R^{5,2} R^{5,3} R^{5,4} R^{5,5} R^{5,6}
R^{6,1} R^{6,2} R^{6,3} R^{6,4} R^{6,5}
R^{7,1} R^{7,2} R^{7,3} R^{7,4}



p1x^{1,1} p1x^{1,2} p1x^{1,k}
(p2x^{1,1})
p4x^{1,1} R^{1,1} R^{1,2} R^{1,k} p4x^{1,k+1} (p2x^{1,k})
p4x^{1,2}
(p3x^{1,1}) p1x^{2,1}
p4x^{2,1} R^{2,1} R^{2,2} R^{1,k} p4x^{2,k+1} (p2x^{2,k})
p4x^{k,1} R^{k,1} R^{k,2} R^{k,k} p4x^{2,k+1} (p2x^{2,k})
p1x^{k+1,1} (p3x^{k,1}) p1x^{k+1,2} (p3x^{k,2}) p1x^{k+1,k} (p3x^{k,k})

Parametric specification of torus

$$
\left(\left(\begin{array}{l}
\left(to_1^{i,j} : pol_1^{i,j}, pb_1^{i,j} -> po_1^{i,j}, pbl^{i,j}\right), \\
\left(ti_{1,v}^{i,j} : pi_1^{i,j}, pbl^{i,j} -> pil_1^{i,j}, pb_v^{i,j}\right), v = \overline{1,4}, v \neq 1, \\
\left(to_3^{i,j} : pil_1^{i+1,j}, pb_3^{i,j} -> pi_1^{i+1,j}, pbl^{i,j}\right), \ \ i \neq k, \\
\left(ti_{3,v}^{i,j} : po_1^{i+1,j}, pbl^{i,j} -> pol_1^{i+1,j}, pb_v^{i,j}\right), \ \ i \neq k, v = \overline{1,4}, v \neq 3, \\
\left(to_4^{i,j} : pol_4^{i,j}, pb_4^{i,j} -> po_4^{i,j}, pbl^{i,j}\right), \\
\left(ti_{4,v}^{i,j} : pi_4^{i,j}, pbl^{i,j} -> pil_4^{i,j}, pb_v^{i,j}\right), v = \overline{1,4}, v \neq 4, \\
\left(to_2^{i,j} : pil_4^{i,j+1}, xb_2^{i,j} -> pi_4^{i,j+1}, pbl^{i,j}\right), \ \ j \neq k, \\
\left(ti_{2,v}^{i,j} : po_4^{i,j+1}, pbl^{i,j} -> pol_4^{i,j+1}, pb_v^{i,j}\right), \ \ j \neq k, v = \overline{1,4}, v \neq 2, \\
\left(to_3^{k,j} : pil_1^{1,j}, pb_3^{k,j} -> pi_1^{1,j}, pbl^{k,j}\right), \\
\left(ti_{3,v}^{k,j} : po_1^{1,j}, pbl^{k,j} -> pol_1^{1,j}, pb_v^{k,j}\right), \ \ v = \overline{1,4}, v \neq 3, \\
\left(to_2^{i,k} : pil_4^{i,1}, xb_2^{i,k} -> pi_4^{i,1}, pbl^{i,k}\right), \\
\left(ti_{2,v}^{i,k} : po_4^{i,1}, pbl^{i,k} -> pol_4^{i,1}, pb_v^{i,k}\right), \ \ v = \overline{1,4}, v \neq 2
\end{array}\right), i = \overline{1,k}, j = \overline{1,k}\right)
$$

Infinite system for p-invariants

$$\begin{cases} to_1^{i,j}: -xpol_1^{i,j} - xpb_1^{i,j} + xpo_1^{i,j} + xpbl^{i,j} = 0, \\ ti_{1,v}^{i,j}: -xpi_1^{i,j} - xpbl^{i,j} + xpil_1^{i,j} + xpb_v^{i,j} = 0, v = 2,3,4, \\ to_3^{i,j}: -xpil_1^{i+1,j} - xpb_3^{i,j} + xpi_1^{i+1,j} + xpbl^{i,j} = 0, \\ ti_{3,v}^{i,j}: -xpo_1^{i+1,j} - xpbl^{i,j} + xpol_1^{i+1,j} + xpb_v^{i,j} = 0, \;\; v = 1,2,4, \\ to_4^{i,j}: -xpol_4^{i,j} - xpb_4^{i,j} + xpo_4^{i,j} + xpbl^{i,j} = 0, \\ ti_{4,v}^{i,j}: -xpi_4^{i,j} - xpbl^{i,j} + xpil_4^{i,j} + xpb_v^{i,j} = 0, v = 1,2,3, \\ to_2^{i,j}: -xpil_4^{i,j+1} - xpb_2^{i,j} + xpi_4^{i,j+1} + pbl^{i,j} = 0, \\ ti_{2,v}^{i,j}: -xpo_4^{i,j+1} - xpbl^{i,j} + xpol_4^{i,j+1} + xpb_v^{i,j} = 0, \;\; v = 1,3,4 \;\; ; i = \overline{1,k}, j = \overline{1,k}. \end{cases}$$

P-invariants of torus

$$\left(\begin{array}{l}
(pi_1^{i,j}, pil_1^{i,j}), i=\overline{1,k}, j=\overline{1,k};\\
(po_1^{i,j}, pol_1^{i,j}), i=\overline{1,k}, j=\overline{1,k};\\
(pi_4^{i,j}, pil_4^{i,j}), i=\overline{1,k}, j=\overline{1,k};\\
(po_4^{i,j}, pol_4^{i,j}), i=\overline{1,k}, j=\overline{1,k};\\
(pb_1^{i,j}, pb_2^{i,j}, pb_3^{i,j}, pb_4^{i,j}, pbl^{i,j}), i=\overline{1,k}, j=\overline{1,k};\\
((pil_1^{i,j}, pol_1^{i,j}, pil_4^{i,j}, pol_4^{i,j}, pbl^{i,j}), i=\overline{1,k}, j=\overline{1,k};)\\
((pi_1^{i,j}, po_1^{i,j}, pi_4^{i,j}, po_4^{i,j}, ((pb_u^{i,j}), u=\overline{1,4};)), i=\overline{1,k}, j=\overline{1,k};)
\end{array}\right)$$

Invariants of 3 x 3 torus

<table>
<tr><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr>
<td>$(pi_1^{1,1}, pil_1^{1,1})$
$(pi_1^{1,2}, pil_1^{1,2})$
$(pi_1^{1,3}, pil_1^{1,3})$
$(pi_1^{2,1}, pil_1^{2,1})$
$(pi_1^{2,2}, pil_1^{2,2})$
$(pi_1^{2,3}, pil_1^{2,3})$
$(pi_1^{3,1}, pil_1^{3,1})$
$(pi_1^{3,2}, pil_1^{3,2})$
$(pi_1^{3,3}, pil_1^{3,3})$</td>
<td>$(po_1^{1,1}, pol_1^{1,1})$
$(po_1^{1,2}, pol_1^{1,2})$
$(po_1^{1,3}, pol_1^{1,3})$
$(po_1^{2,1}, pol_1^{2,1})$
$(po_1^{2,2}, pol_1^{2,2})$
$(po_1^{2,3}, pol_1^{2,3})$
$(po_1^{3,1}, pol_1^{3,1})$
$(po_1^{3,2}, pol_1^{3,2})$
$(po_1^{3,3}, pol_1^{3,3})$</td>
<td>$(pi_4^{1,1}, pil_4^{1,1})$
$(pi_4^{1,2}, pil_4^{1,2})$
$(pi_4^{1,3}, pil_4^{1,3})$
$(pi_4^{2,1}, pil_4^{2,1})$
$(pi_4^{2,2}, pil_4^{2,2})$
$(pi_4^{2,3}, pil_4^{2,3})$
$(pi_4^{3,1}, pil_4^{3,1})$
$(pi_4^{3,2}, pil_4^{3,2})$
$(pi_4^{3,3}, pil_4^{3,3})$</td>
<td>$(po_4^{1,1}, pol_4^{1,1})$
$(po_4^{1,2}, pol_4^{1,2})$
$(po_4^{1,3}, pol_4^{1,3})$
$(po_4^{2,1}, pol_4^{2,1})$
$(po_4^{2,2}, pol_4^{2,2})$
$(po_4^{2,3}, pol_4^{2,3})$
$(po_4^{3,1}, pol_4^{3,1})$
$(po_4^{3,2}, pol_4^{3,2})$
$(po_4^{3,3}, pol_4^{3,3})$</td>
<td>$(pb_1^{1,1}, pb_2^{1,1}, pb_3^{1,1}, \mathrm{pb}_4^{1,1}, pbl^{1,1})$
$(pb_1^{1,2}, pb_2^{1,2}, pb_3^{1,2}, \mathrm{pb}_4^{1,2}, pbl^{1,2})$
$(pb_1^{1,3}, pb_2^{1,3}, pb_3^{1,3}, \mathrm{pb}_4^{1,3}, pbl^{1,3})$
$(pb_1^{2,1}, pb_2^{2,1}, pb_3^{2,1}, \mathrm{pb}_4^{2,1}, pbl^{2,1})$
$(pb_1^{2,2}, pb_2^{2,2}, pb_3^{2,2}, \mathrm{pb}_4^{2,2}, pbl^{2,2})$
$(pb_1^{2,3}, pb_2^{2,3}, pb_3^{2,3}, \mathrm{pb}_4^{2,3}, pbl^{2,3})$
$(pb_1^{3,1}, pb_2^{3,1}, pb_3^{3,1}, \mathrm{pb}_4^{3,1}, pbl^{3,1})$
$(pb_1^{3,2}, pb_2^{3,2}, pb_3^{3,2}, \mathrm{pb}_4^{3,2}, pbl^{3,2})$
$(pb_1^{3,3}, pb_2^{3,3}, pb_3^{3,3}, \mathrm{pb}_4^{3,3}, pbl^{3,3})$</td>
</tr>
<tr><th colspan="5">6</th></tr>
<tr><td colspan="5">$(pil_1^{1,1}, pol_1^{1,1}, pil_4^{1,1}, pol_4^{1,1}, pbl^{1,1}, pil_1^{1,2}, pol_1^{1,2}, pil_4^{1,2}, pol_4^{1,2}, pbl^{1,2}, pil_1^{1,3}, pol_1^{1,3},$
$pil_4^{1,3}, pol_4^{1,3}, pbl^{1,3}, pil_1^{2,1}, pol_1^{2,1}, pil_4^{2,1}, pol_4^{2,1}, pbl^{2,1}, pil_1^{2,2}, pol_1^{2,2}, pil_4^{2,2},$
$pol_4^{2,2}, pbl^{2,2}, pil_1^{2,3}, pol_1^{2,3}, pil_4^{2,3}, pol_4^{2,3}, pbl^{2,3}, pil_1^{3,1}, pol_1^{3,1}, pil_4^{3,1}, pol_4^{3,1},$
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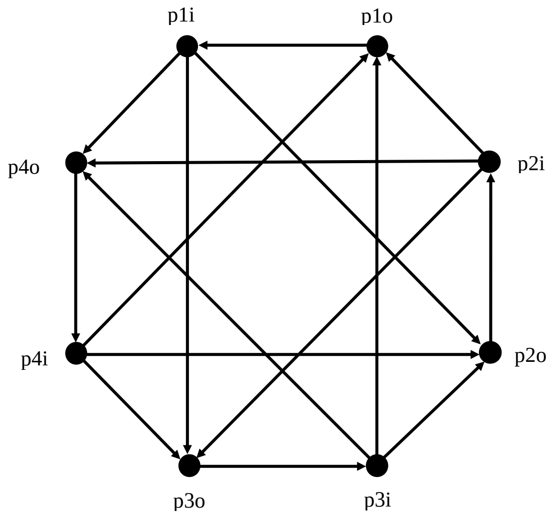
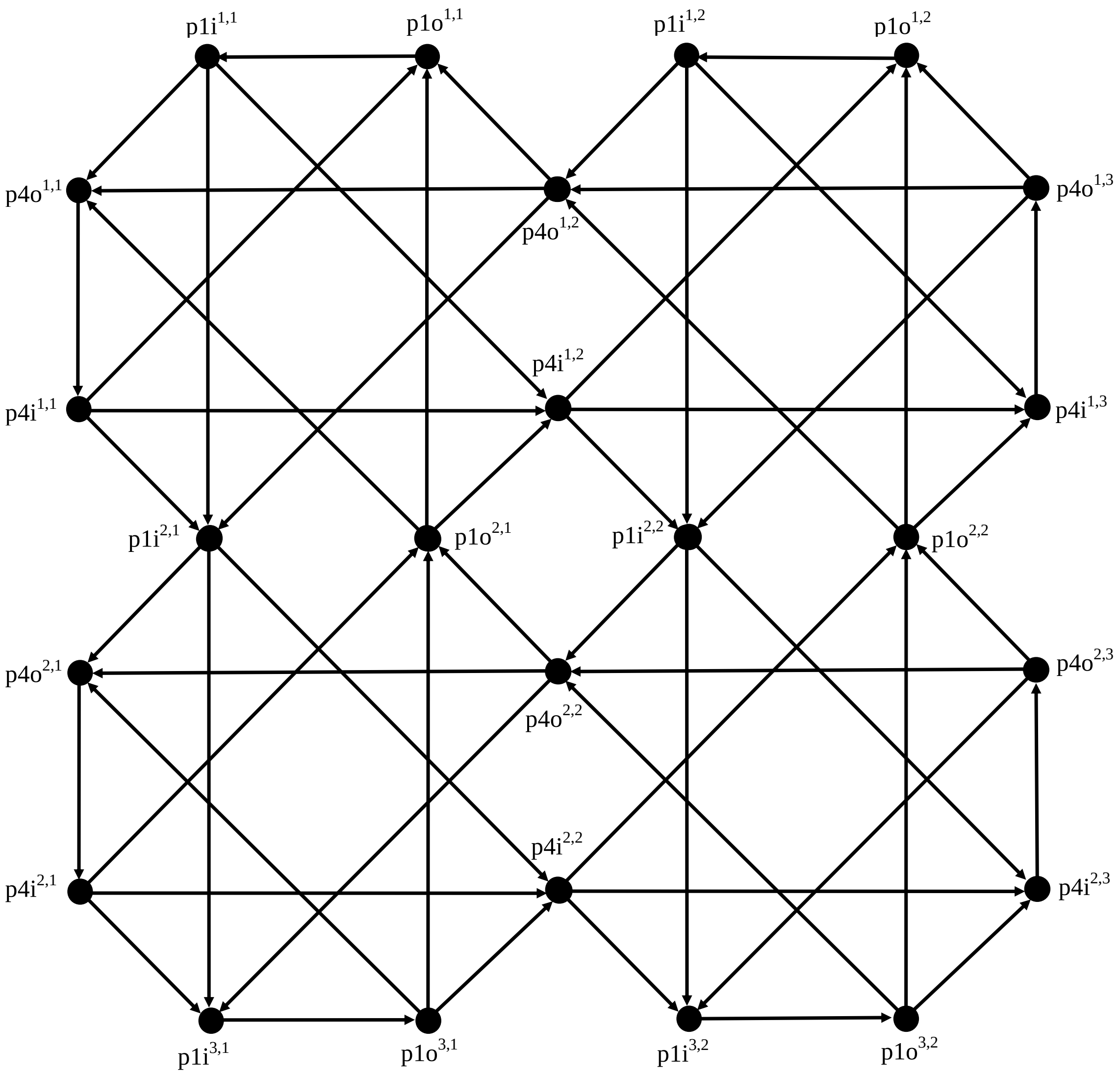
<tr><th colspan="5">7</th></tr>
<tr><td colspan="5">$(pi_1^{1,1}, po_1^{1,1}, pi_4^{1,1}, po_4^{1,1}, pb_1^{1,1}, pb_2^{1,1}, pb_3^{1,1}, pb_4^{1,1}, pi_1^{1,2}, po_1^{1,2}, pi_4^{1,2}, po_4^{1,2},$
$pb_1^{1,2}, pb_2^{1,2}, pb_3^{1,2}, pb_4^{1,2}, pi_1^{1,3}, po_1^{1,3}, pi_4^{1,3}, po_4^{1,3}, pb_1^{1,3}, pb_2^{1,3}, pb_3^{1,3}, pb_4^{1,3},$
$pi_1^{2,1}, po_1^{2,1}, pi_4^{2,1}, po_4^{2,1}, pb_1^{2,1}, pb_2^{2,1}, pb_3^{2,1}, pb_4^{2,1}, pi_1^{2,2}, po_1^{2,2}, pi_4^{2,2}, po_4^{2,2},$
$pb_1^{2,2}, pb_2^{2,2}, pb_3^{2,2}, pb_4^{2,2}, pi_1^{2,3}, po_1^{2,3}, pi_4^{2,3}, po_4^{2,3}, pb_1^{2,3}, pb_2^{2,3}, pb_3^{2,3}, pb_4^{2,3},$
$pi_1^{3,1}, po_1^{3,1}, pi_4^{3,1}, po_4^{3,1}, pb_1^{3,1}, pb_2^{3,1}, pb_3^{3,1}, pb_4^{3,1}, pi_1^{3,2}, po_1^{3,2}, pi_4^{3,2}, po_4^{3,2},$
$pb_1^{3,2}, pb_2^{3,2}, pb_3^{3,2}, pb_4^{3,2}, pi_1^{3,3}, po_1^{3,3}, pi_4^{3,3}, po_4^{3,3}, pb_1^{3,3}, pb_2^{3,3}, pb_3^{3,3}, pb_4^{3,3})$</td></tr>
</table>

Method of grids analysis

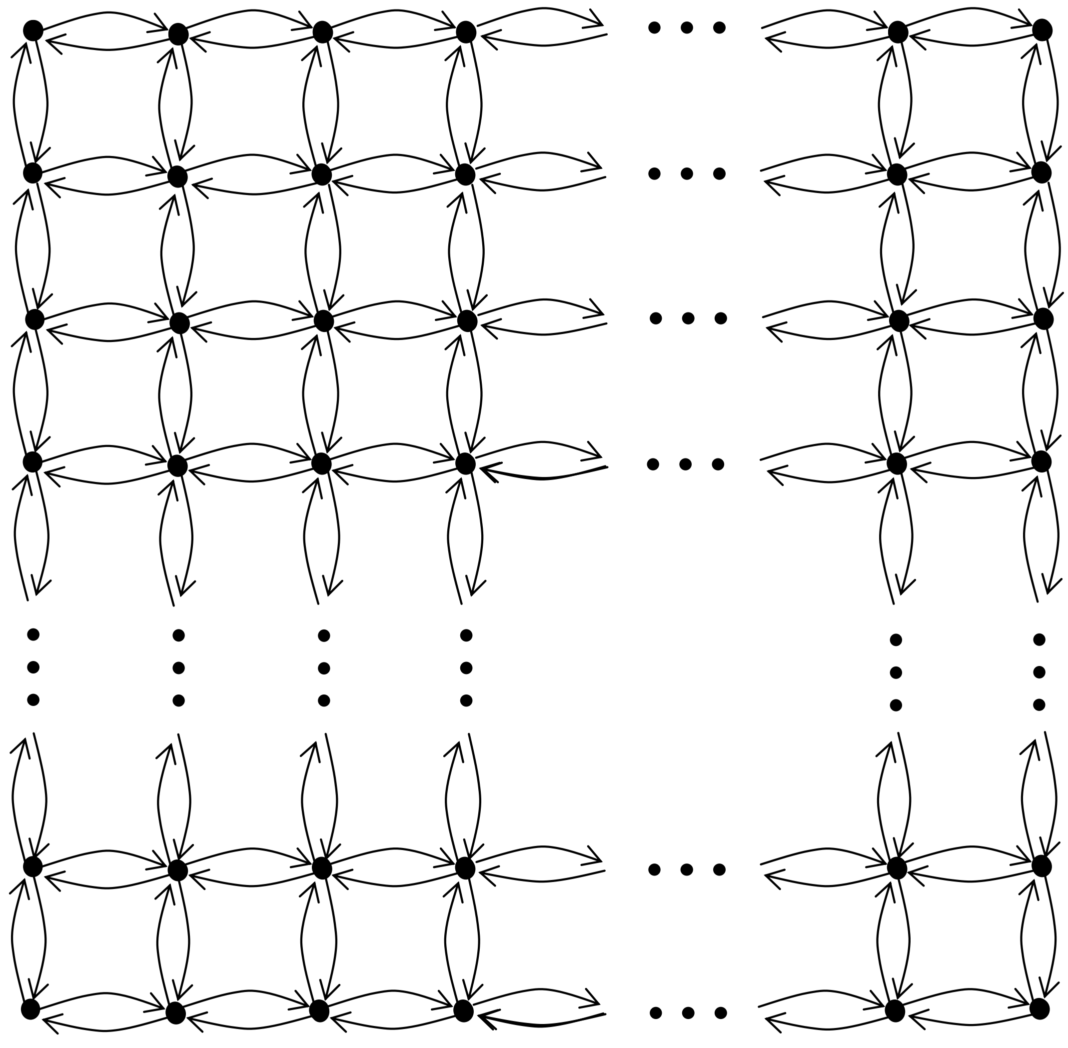
- Solving infinite systems of linear algebraic equations for p- and t-invariants
- Implicit construction of cyclic sequences of transitions' firing
- Auxiliary graphs of packets' transmission and possible blockings of devices

Graph of transmissions

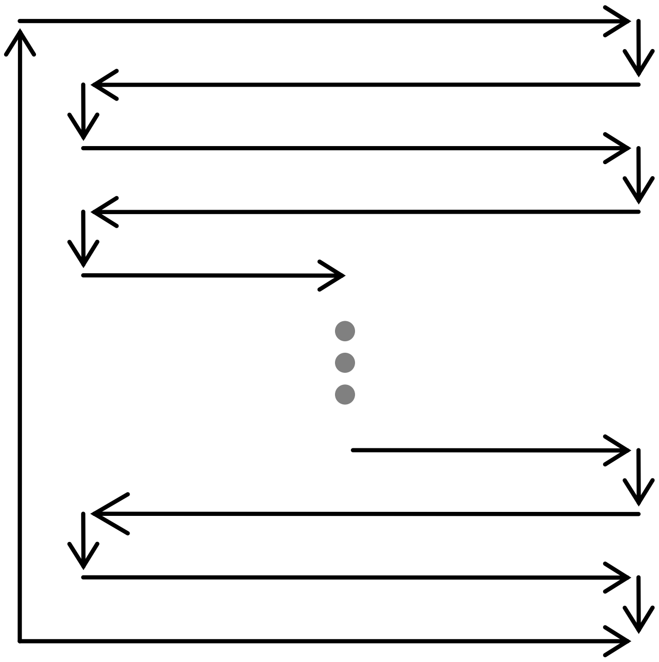


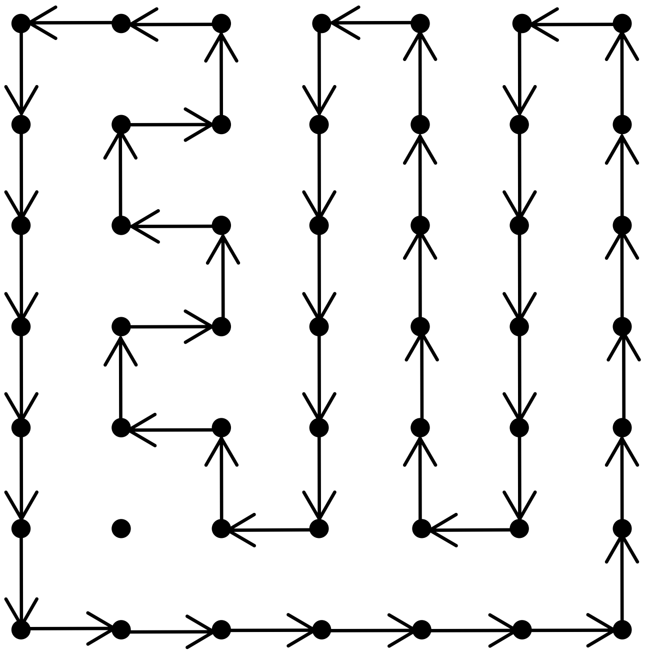


Graph of possible blockings



Detours of possible blockings graphs

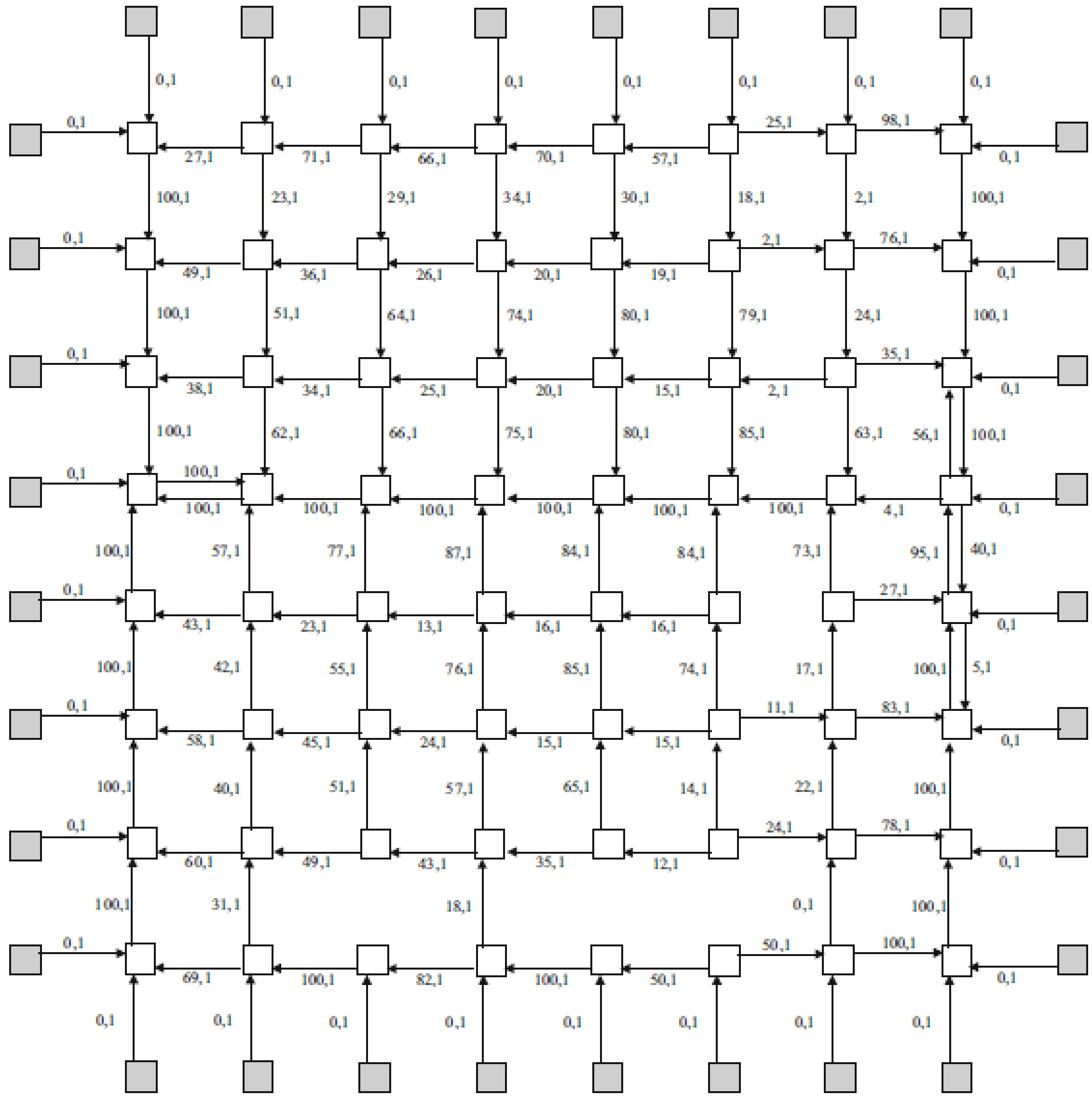




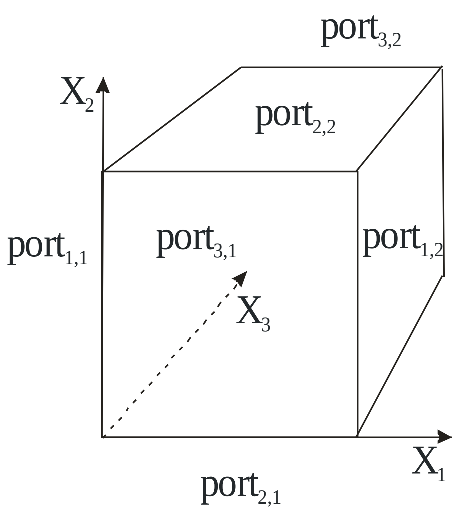
Complex deadlocks

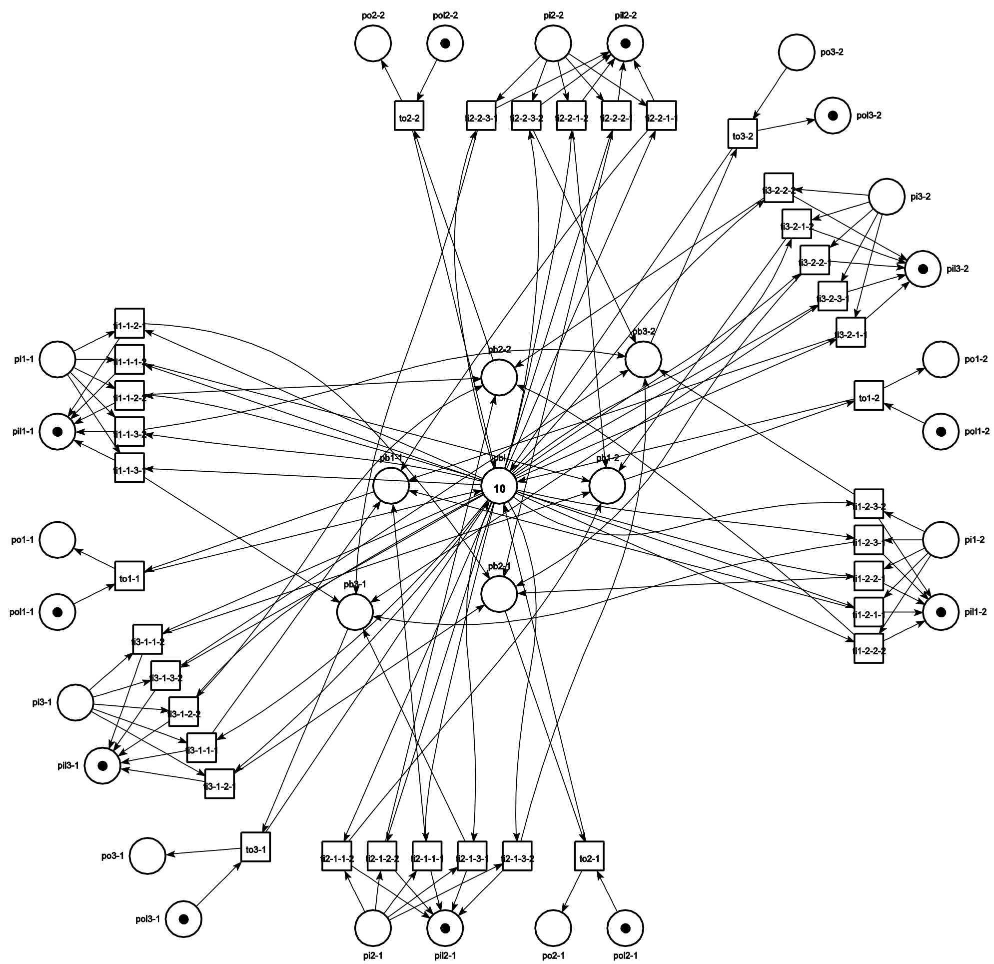
- Circle of blockings
- Isolation of a vertex
- Chain of blockings with its end on an earlier blocked chain
- Avalanche-like growth of the blocked nodes number
- Possibility of blocking a network by ill-intentioned traffic

Example of deadlock

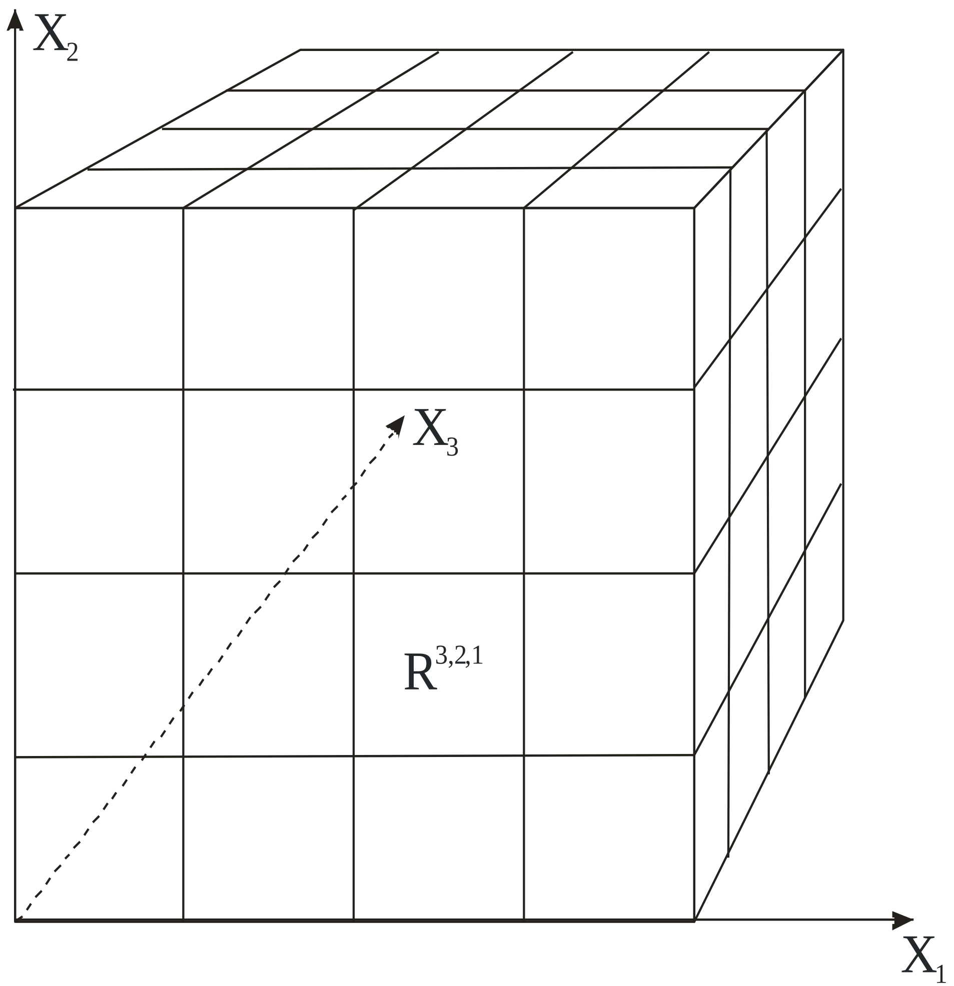


Cubic device

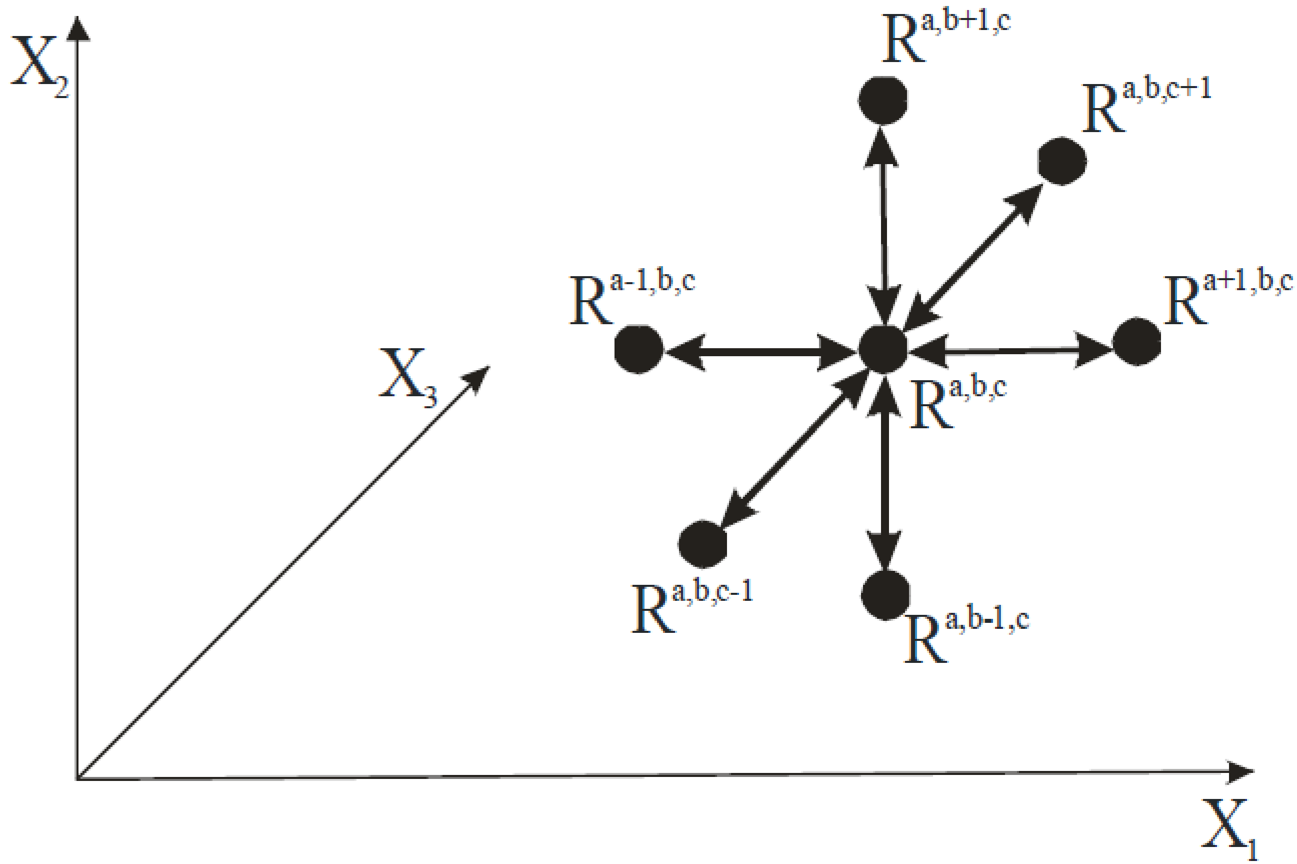




Composition of a cube



Neighbors of a cube



Hypercube composition

$$\left(\left(\begin{array}{l}\left(ti^{i_1,i_2,\ldots i_d}_{j,n,j',n'} : pi^{i_1,i_2,\ldots i_d}_{j,n}, pbl^{i_1,i_2,\ldots i_d} \to pb^{i_1,i_2,\ldots i_d}_{j',n'}, pil^{i_1,i_2,\ldots i_d}_{j,n}\right), \\ j' = \overline{1,d},\quad n' = 1,2,\quad j' \neq j,\quad n' \neq n; \\ \left(to^{i_1,i_2,\ldots i_d}_{j,n} : pb^{i_1,i_2,\ldots i_d}_{j,n}, pol^{i_1,i_2,\ldots i_d}_{j,n} \to po^{i_1,i_2,\ldots i_d}_{j,n}, pbl^{i_1,i_2,\ldots i_d}\right)\end{array}\right), j = \overline{1,d}, n = \overline{1,2}\right)$$

$$\left(\left(\begin{array}{l} pi^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} := po^{i_1,\ldots,i_j,\ldots i_d}_{j,2} \cup pi^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} \\ pil^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} := pol^{i_1,\ldots,i_j,\ldots i_d}_{j,2} \cup pil^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} \\ po^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} := pi^{i_1,\ldots,i_j,\ldots i_d}_{j,2} \cup po^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} \\ pol^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} := pil^{i_1,\ldots,i_j,\ldots i_d}_{j,2} \cup pol^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} \end{array}\right), i_u = \overline{1,k-1}, u = \overline{1,d}, j = \overline{1,d}\right)$$

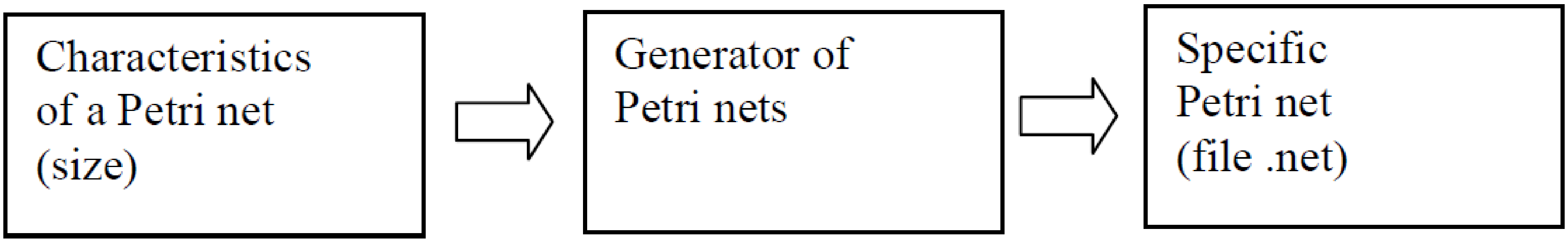
$$\left(\left(\begin{array}{l} pi^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} := po^{i_1,\ldots,i_j,\ldots i_d}_{j,2} \\ pil^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} := pol^{i_1,\ldots,i_j,\ldots i_d}_{j,2} \\ po^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} := pi^{i_1,\ldots,i_j,\ldots i_d}_{j,2} \\ pol^{i_1,\ldots,i_j+1,\ldots i_d}_{j,1} := pil^{i_1,\ldots,i_j,\ldots i_d}_{j,2} \end{array}\right), i_u = \overline{1,k-1}, u = \overline{1,d}, j = \overline{1,d}, u \neq j, i_j = k\right)$$

P-invariants of hypercube

$$\begin{cases}
to_{j,1}^{i_1,\dots,i_d}: \quad xpb_{j,1}^{i_1,\dots,i_d} + xpol_{j,1}^{i_1,\dots,i_d} = xpo_{j,1}^{i_1,\dots,i_d} + xpbl^{i_1,\dots,i_d}, \\
ti_{j,1,j',n'}^{i_1,\dots,i_d}: \quad xpi_{j,1}^{i_1,\dots,i_d} + xpbl^{i_1,\dots,i_d} = xpb_{j',n'}^{i_1,\dots,i_d} + xpil_{j,1}^{i_1,\dots,i_d}, \\
to_{j,2}^{i_1,\dots,i_j,\dots,i_d}: \quad xpb_{j,2}^{i_1,\dots,i_j,\dots,i_d} + xpil_{j,1}^{i_1,\dots,i_j+1,\dots,i_d} = xpi_{j,1}^{i_1,\dots,i_j+1,\dots,i_d} + xpbl^{i_1,\dots,i_j,\dots,i_d}, \\
ti_{j,2,j',n'}^{i_1,\dots,i_j,\dots,i_d}: \quad xpo_{j,1}^{i_1,\dots,i_j+1,\dots,i_d} + xpbl^{i_1,\dots,i_j,\dots,i_d} = xpb_{j',n'}^{i_1,\dots,i_j,\dots,i_d} + xpol_{j,1}^{i_1,\dots,i_j+1,\dots,i_d}, \\
\qquad j=\overline{1,d}, \quad j'=\overline{1,d}, \quad n'=1,2, \quad j' \neq j, \quad n' \neq n, \quad i_u=\overline{1,k}, \quad u=\overline{1,d}.
\end{cases}$$

$$\left(\begin{array}{l}
(pi_{j,1}^{i_1,\dots,i_j,\dots,i_d}, pil_{j,1}^{i_1,\dots,i_j,\dots,i_d}), \quad j=\overline{1,d}, \quad (i_u=\overline{1,k}, \quad u=\overline{1,d}, \quad u \neq j), \quad i_j=\overline{1,k+1}; \\
(po_{j,1}^{i_1,\dots,i_j,\dots,i_d}, pol_{j,1}^{i_1,\dots,i_j,\dots,i_d}), \quad j=\overline{1,d}, \quad (i_u=\overline{1,k}, \quad u=\overline{1,d}, \quad u \neq j), \quad i_j=\overline{1,k+1}; \\
(pbl^{i_1,\dots,i_d}, (pb_{j,n}^{i_1,\dots,i_d}, j=\overline{1,d}, \quad n=1,2)), \quad i_u=\overline{1,k}, \quad u=\overline{1,d}; \\
((pb_{j,n}^{i_1,\dots,i_d}, n=1,2, \ j=\overline{1,d}, i_u=\overline{1,k}, u=\overline{1,d}), \quad ((pi_{j,1}^{i_1,\dots,i_d}, po_{j,1}^{i_1,\dots,i_d}), j=\overline{1,d}, i_u=\overline{1,k}, u=\overline{1,d}), \\
\qquad ((pi_{j,1}^{i_1,\dots,i_j,\dots,i_d}, po_{j,1}^{i_1,\dots,i_j,\dots,i_d}), j=\overline{1,d}, \quad i_u=\overline{1,k}, \quad u=\overline{1,d}, \quad u \neq j, \quad i_j=k+1)) \\
((pbl^{i_1,\dots,i_d}, ((pil_{j,1}^{i_1,\dots,i_d}, pol_{j,1}^{i_1,\dots,i_d}), j=\overline{1,d}), \quad i_u=\overline{1,k}, \quad u=\overline{1,d}), \\
\qquad ((pil_{j,1}^{i_1,\dots,i_j,\dots,i_d}, pol_{j,1}^{i_1,\dots,i_j,\dots,i_d}), j=\overline{1,d}, \quad i_u=\overline{1,k}, \quad u=\overline{1,d}, \quad u \neq j, \quad i_j=k+1))
\end{array}\right)$$

Generators of models

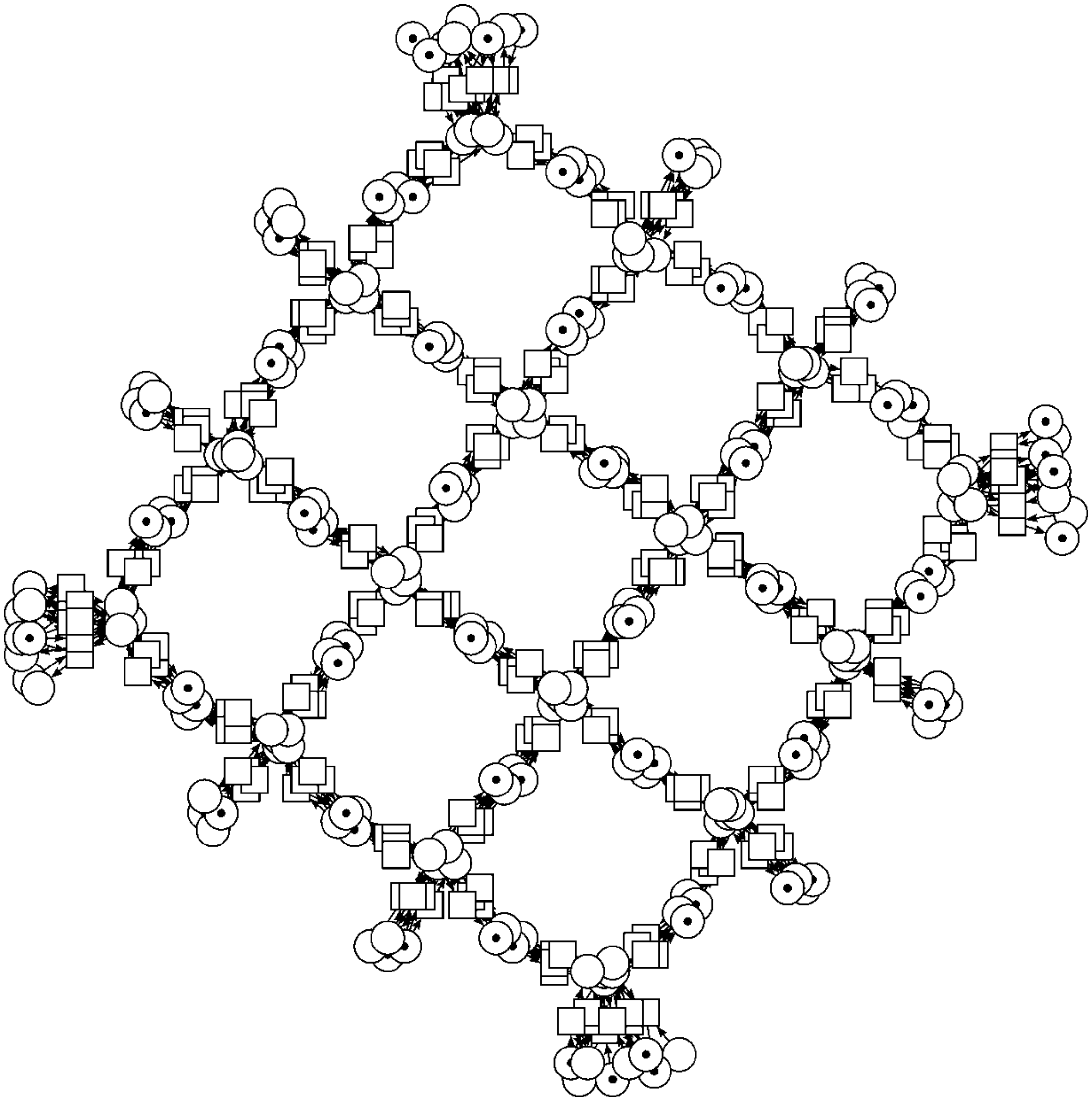


- Square grid (graphical form)
- Hypercube grid
- Hypertorus grid
- Canvas of generalized neigborhood
- Computers of double exponent

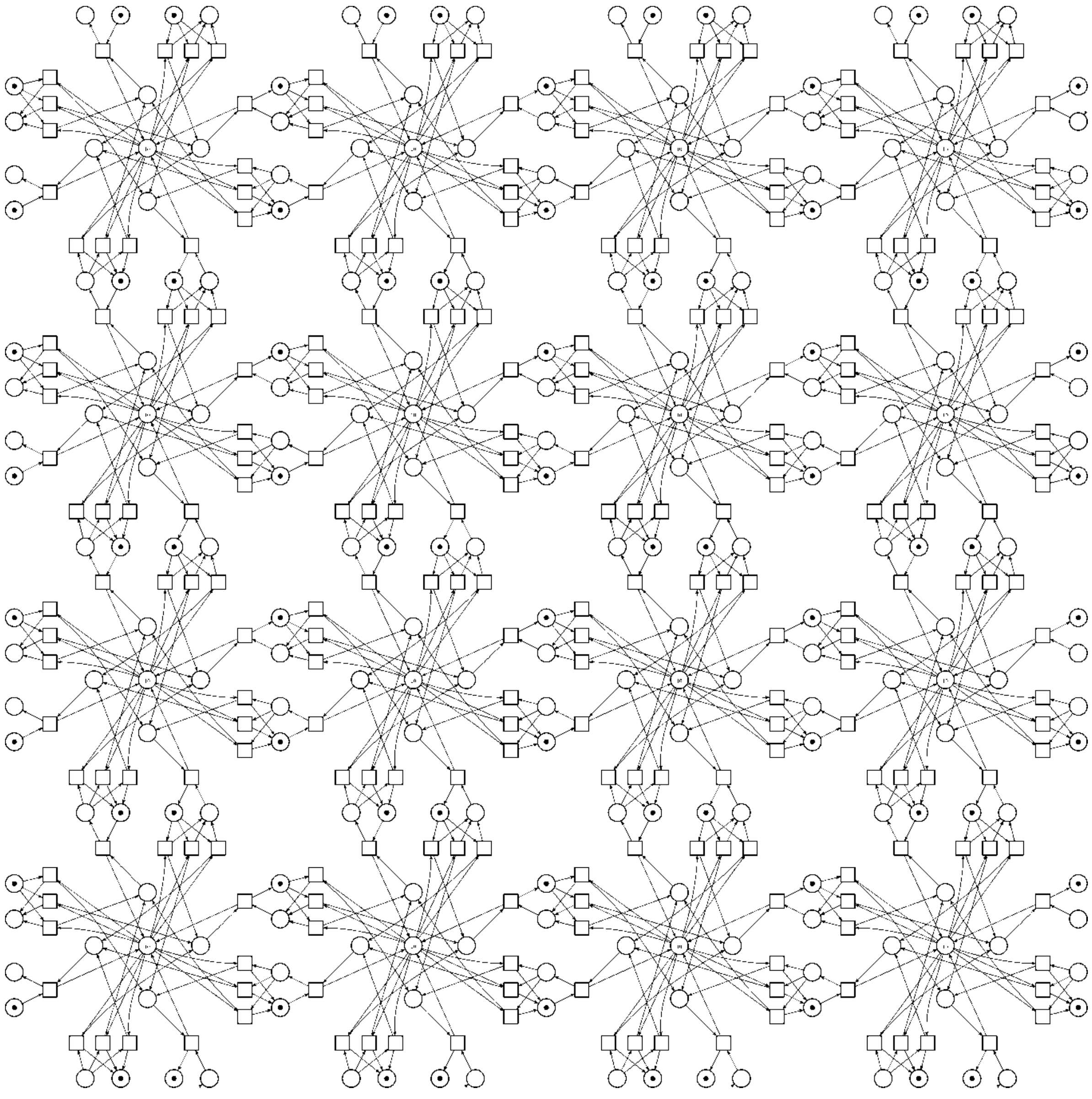
Generator of an open grid

```
main( int argc, char * argv[] )
{
 int i, j, k = atoi( argv[1] );
 for(i=1; i<=k; i++)
   for(j=1; j<=k; j++)  {
     printf("tr {to_1^%d,%d} {pol_1^%d,%d} {pb_1^%d,%d} -> {po_1^%d,%d} {pbl^%d,%d}\n", i,j, i,j, i,j, i,j, i,j );
     printf("tr {ti_1,2^%d,%d} {pi_1^%d,%d} {pbl^%d,%d} -> {pil_1^%d,%d} {pb_2^%d,%d}\n", i,j, i,j, i,j, i,j, i,j );
     printf("tr {ti_1,3^%d,%d} {pi_1^%d,%d} {pbl^%d,%d} -> {pil_1^%d,%d} {pb_3^%d,%d}\n", i,j, i,j, i,j, i,j, i,j );
     printf("tr {ti_1,4^%d,%d} {pi_1^%d,%d} {pbl^%d,%d} -> {pil_1^%d,%d} {pb_4^%d,%d}\n", i,j, i,j, i,j, i,j, i,j );
     printf("tr {to_4^%d,%d} {pol_4^%d,%d} {pb_4^%d,%d} -> {po_4^%d,%d} {pbl^%d,%d}\n", i,j, i,j, i,j, i,j, i,j );
     printf("tr {ti_4,1^%d,%d} {pi_4^%d,%d} {pbl^%d,%d} -> {pil_4^%d,%d} {pb_1^%d,%d}\n", i,j, i,j, i,j, i,j, i,j );
     printf("tr {ti_4,2^%d,%d} {pi_4^%d,%d} {pbl^%d,%d} -> {pil_4^%d,%d} {pb_2^%d,%d}\n", i,j, i,j, i,j, i,j, i,j );
     printf("tr {ti_4,3^%d,%d} {pi_4^%d,%d} {pbl^%d,%d} -> {pil_4^%d,%d} {pb_3^%d,%d}\n", i,j, i,j, i,j, i,j, i,j );
     printf("tr {to_2^%d,%d} {pil_4^%d,%d} {pb_2^%d,%d} -> {pi_4^%d,%d} {pbl^%d,%d}\n", i,j, i,j+1, i,j, i,j+1, i,j );
     printf("tr {ti_2,1^%d,%d} {po_4^%d,%d} {pbl^%d,%d} -> {pol_4^%d,%d} {pb_1^%d,%d}\n", i,j, i,j+1, i,j, i,j+1, i,j );
     printf("tr {ti_2,3^%d.%d} {po_4^%d,%d} {pbl^%d,%d} -> {pol_4^%d,%d} {pb_3^%d,%d}\n", i,j, i,j+1, i,j, i,j+1, i,j );
     printf("tr {ti_2,4^%d,%d} {po_4^%d,%d} {pbl^%d,%d} -> {pol_4^%d,%d} {pb_4^%d,%d}\n", i,j, i,j+1, i,j, i,j+1, i,j );
     printf("tr {to_3^%d,%d} {pil_1^%d,%d} {pb_3^%d,%d} -> {pi_1^%d,%d} {pbl^%d,%d}\n", i,j, i+1,j, i,j, i+1,j, i,j );
     printf("tr {ti_3,1^%d,%d} {po_1^%d^%d} {pbl^%d,%d} -> {pol_1^%d,%d} {pb_1^%d,%d}\n", i,j, i+1,j, i,j, i+1,j, i,j);
     printf("tr {ti_3,2^%d,%d} {po_1^%d,%d} {pbl^%d,%d} -> {pol_1^%d,%d} {pb_2^%d,%d}\n", i,j, i+1,j, i,j, i+1,j, i,j );
     printf("tr {ti_3,4^%d,%d} {po_1^%d,%d} {pbl^%d,%d} -> {pol_1^%d,%d} {pb_4^%d,%d}\n", i,j, i+1,j, i,j, i+1,j, i,j );
   }
 printf("net n2o%d\n", k);
}
```

Automatic visualization

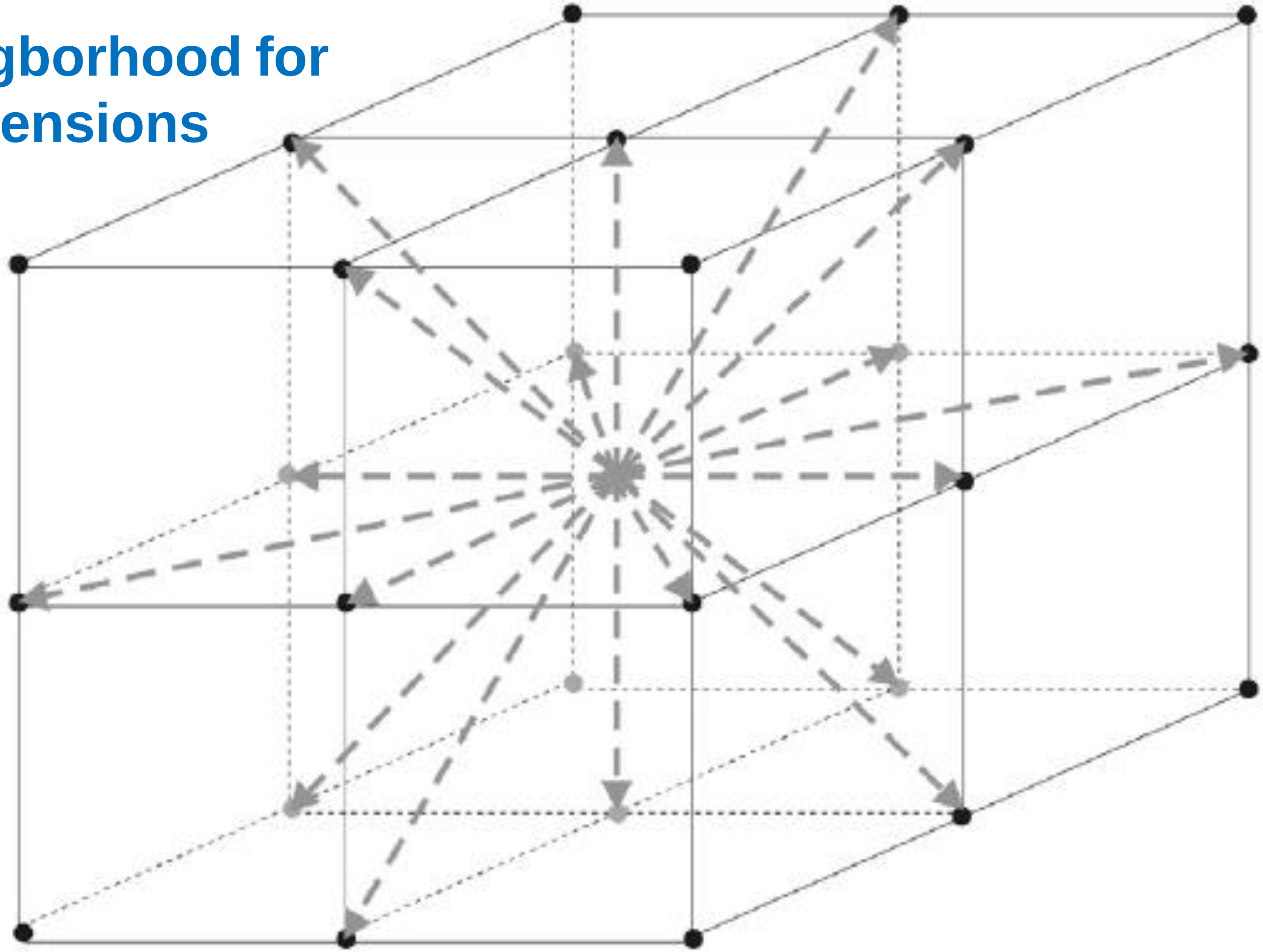


Generated graphical format



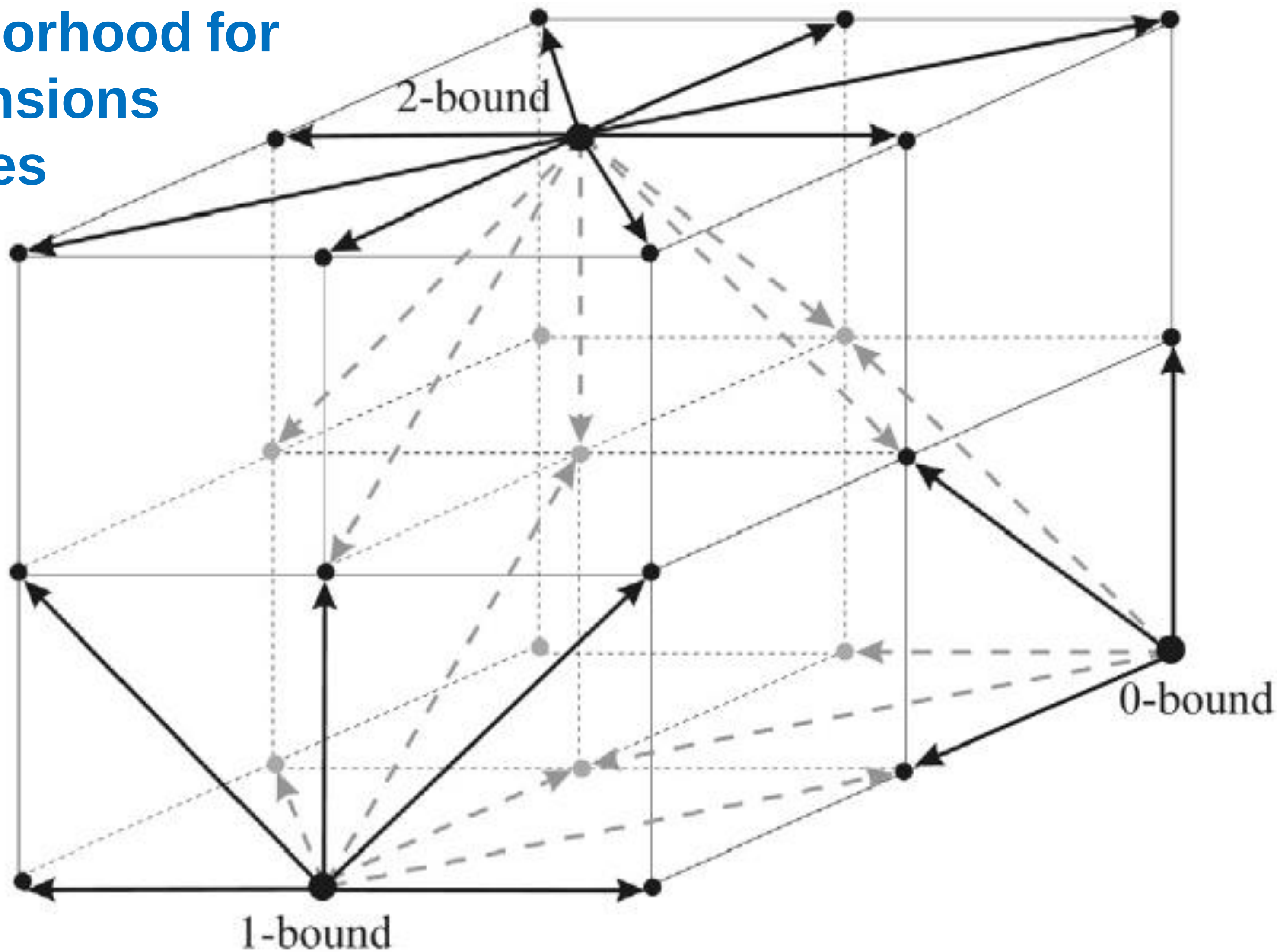
Generalized neighborhoods

2-neigborhood for 3 dimensions

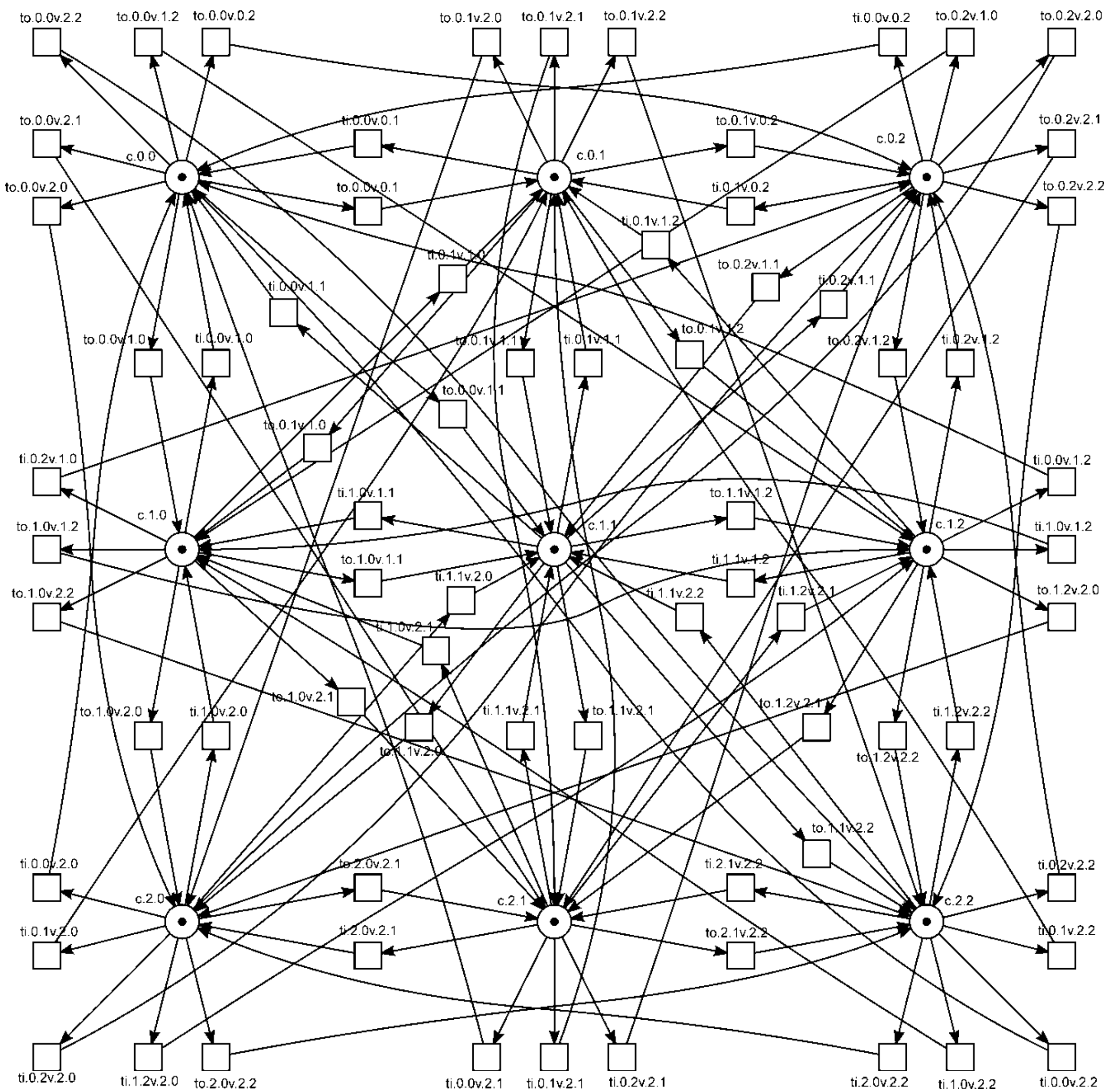


Generalized neighborhoods

2-neigborhood for 3 dimensions on edges



Moore neighborhood in a torus



Theorems and Hypothesis

Theorem 1. Model of a hypercube is a p-invariant Petri net for arbitrary natural numbers *d, k*.

Theorem 2. Model of a hypercube is a t-invariant Petri net for arbitrary natural numbers *d, k*.

Statement. Model of a hypercube is not a live Petri net (it contains deadlocks).

Hypothesis1. Appearing deadlocks in the grid models increase probability of appearing new deadlocks.

Hypothesis 2. Process of deadlocks creation can be controlled by generators of (perilous) traffic.

Basic publications

- **Zaitsev D.A., Zaitsev I.D. and Shmeleva T.R. Infinite Petri Nets: Part 1, Modeling Square Grid Structures, Complex Systems, 26(2), 2017, 157-195.**
- **Zaitsev D.A., Zaitsev I.D. and Shmeleva T.R. Infinite Petri Nets: Part 2, Modeling Triangular, Hexagonal, Hypercube and Hypertorus Structures, Complex Systems, 26(4), 2017, forthcoming**
- Zaitsev D.A. A generalized neighborhood for cellular automata, Theoretical Computer Science, 666 (2017), 21–35.
- Zaitsev D.A. Verification of Computing Grids with Special Edge Conditions by Infinite Petri Nets, Automatic Control and Computer Sciences, 2013, Vol. 47, No. 7, pp. 403–412.
- Zaitsev D.A., Shmeleva T.R. Verification of hypercube communication structures via parametric Petri nets, Cybernetics and Systems Analysis, Volume 46, Number 1 (2010), 105-114
- Shmeleva, T.R., Zaitsev, D.A., Zaitsev, I.D. Verification of square communication grid protocols via infinite Petri nets, MESM 2009 - 10th Middle Eastern Simulation Multiconference, p. 53-59.

Conclusions

- Finite parametric specification of infinite Petri nets
- Construction and solving infinite systems in parametric form
- Proof of infinite Petri net invariance
- Representation of complex deadlocks
- Analysis of computing grids (plane multiangle, hypercube)

Research directions

- Formal methods for solving infinite systems
- Methods for liveness investigation
- Methods of boundedness, conservativeness, liveness enforcing
- Behavioral properties of models
- Composition of a few basic elements on a given pattern

Infinite Petri Nets

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http://daze.ho.ua

http://icl.utk.edu