Verification of protocol TCP via decomposition of Petri net model into functional subnets

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ABSTRACT

Proof of the invariance of Petri net model for connection and disconnection phases of TCP protocol was implemented. Decomposition of Petri net model into functional subnets was realized. Calculation of invariants was implemented in the process of sequential composition, which allows the essential acceleration of computations.

KEYWORDS: TCP, verification, Petri net, invariant, decomposition, functional subnet

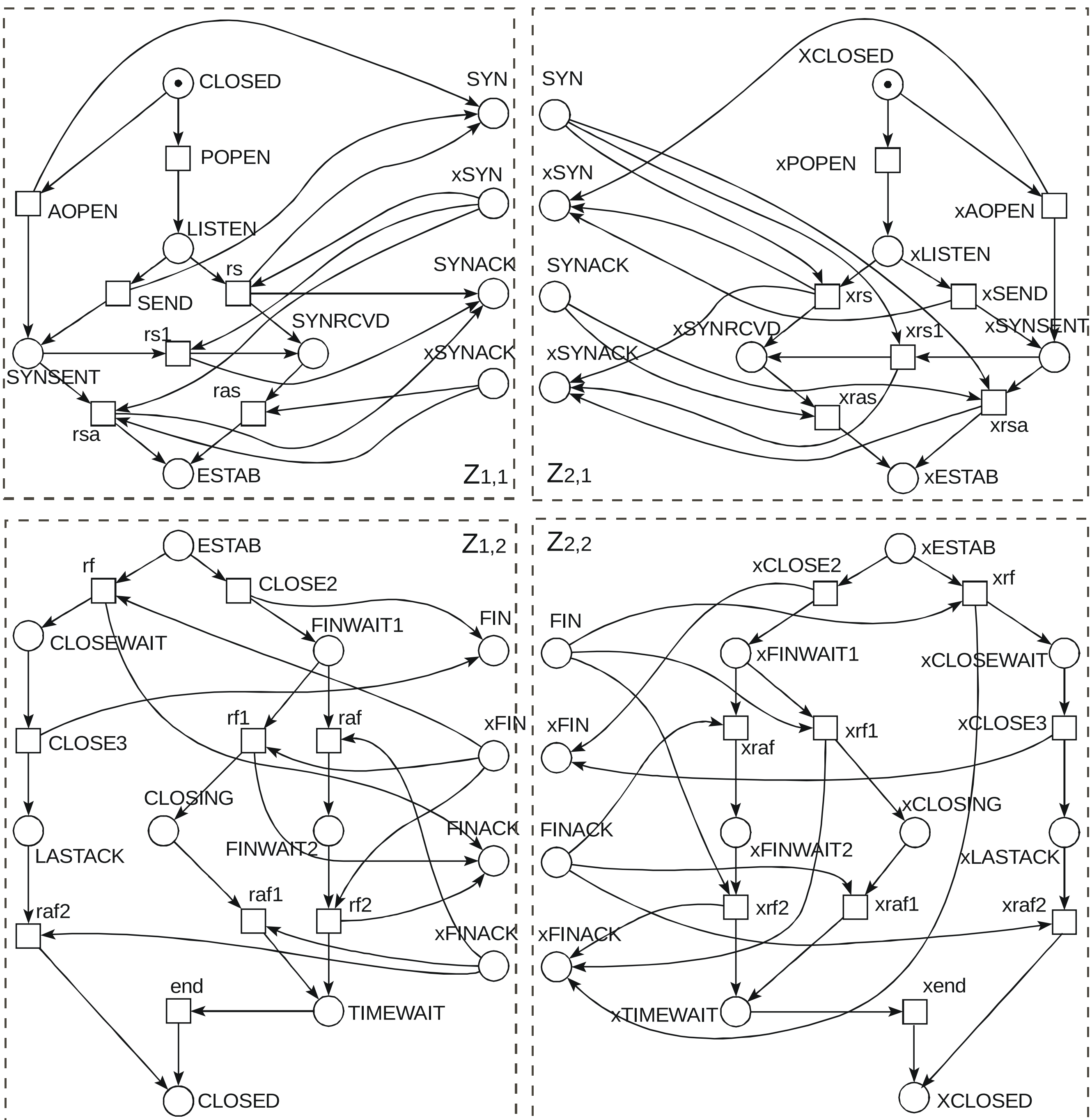

Fig. 1. Decomposition of protocol TCP Petri net model

DECOMPOSITION OF PETRI NET MODEL

Petri net model of protocol TCP was created on the standard specification represented in RFC 793 [9]. The decomposition of protocol TCP model into its minimal functional subnets (Fig.1) according to algorithm [4,5] consists of four minimal functional subnets $\{Z^{1,1}, Z^{1,2}, Z^{2,1}, Z^{2,2}\}$.

INVARIANCE OF MODEL

Invariants [1] are a powerful tool for investigation of structural properties of Petri nets. They allow the determination of boundness, safeness, and necessary conditions of liveness and absence of deadlocks. These properties are significant for real-life systems behavior analysis, especially, for telecommunication protocols [2,3]. Let us implement the technique of decomposition-based calculation of invariants [5,6,7,8] for Petri net model of protocol TCP (Fig. 1).

Let's remind, that Petri net invariant [5] is nonnegative integer solutions $\bar{x}$ of system

$$\bar{x} \cdot A = 0, \qquad (1)$$

where A is the incidence matrix of Petri net for place invariants (p-invariants) or transposed incidence matrix for transition invariants (t-invariants).

According to [5], to calculate invariants of Petri net we should to calculate invariants of its minimal functional subnets and then to find common invariants of contact places.

Let's the general solution for invariant of functional subnet Z^j is represented in the form

$$\bar{x} = \bar{z}^j \cdot G^j, \qquad (2)$$

where $\bar{z}^j$ is an arbitrary vector of nonnegative integer numbers and G^j is a matrix of basis solutions. Then the system of equations for calculation of common invariants of contact places has the form

$$\left\{ \bar{z}^i \cdot G_p^i - \bar{z}^j \cdot G_p^j = 0, \quad p \in C \right., \qquad (3)$$

where $i.j$ are the numbers of functional subnets incidental to place $p \in C$ and G_p^j is the column of matrix G^j corresponding to place p.

Thus, variables $\bar{z}^j$ become non-free now. Notice that, system (3) has the same form as the source system (1). Consequently, for its solution we may apply the same methods. Let's assume that $\bar{z} = \bar{y} \cdot R$, where R is a matrix of basis solutions of system (3) and $\bar{y}$ consists of arbitrary nonnegative integer numbers. Then the general solution of system (1) according to (2) may be represented as

$$\bar{x} = \bar{y} \cdot H, \; H = R \cdot G. \qquad (4)$$

In the cases the model possesses the internal symmetry owing to some minimal functional subnets are isomorphic the process described is advisable to execute in a sequential way. We use isomorphism of subnets Z^1 and Z^2: at the beginning we calculate invariants of subnet Z^1, then construct invariant of isomorphic subnet Z^2 and finally calculate the invariant of entire given Petri net.

Table 1. Places of net

№	Name	№	Name	№	Name
1	CLOSED	11	TIMEWAIT	21	xLISTEN
2	LISTEN	12	SYN	22	xSYNSENT
3	SYNSENT	13	xSYN	23	xSYNRCVD
4	SYNRCVD	14	SYNACK	24	xESTAB
5	ESTAB	15	xSYNACK	25	XCLOSEWAIT
6	CLOSEWAIT	16	FIN	26	xFINWAIT1
7	FINWAIT1	17	xFIN	27	XLASTACK
8	LASTACK	18	FINACK	28	XCLOSING
9	CLOSING	19	xFINACK	29	xFINWAIT2
10	FINWAIT2	20	xCLOSED	30	XTIMEWAIT

Basis invariants of subnets $Z^{1,1}$ and $Z^{1,2}$ calculated with the aid of Toudic algorithm [10-12] with respect to numeration of places defined by Table 1 may be represented in a matrix form with respect to vectors

$$(x_1, x_2, x_3, x_4, x_5, x_{12}, x_{13}, x_{14}, x_{15}) =$$
$$(z_1^1, z_2^1, z_3^1, z_4^1, z_5^1, z_6^1) \cdot G^{1,1},$$
$$(x_1, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{16}, x_{17}, x_{18}, x_{19}) =$$
$$(z_1^2, z_2^2, z_3^2, z_4^2, z_5^2, z_6^2) \cdot G^{1,2},$$

where the matrixes $G^{1,1}$ and $G^{1,2}$ have the form

$$G^{1,1} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{vmatrix},$$

$$G^{1,2} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{vmatrix}.$$

Notice that, components of vector $\bar{x}$ corresponding to subnets $Z^{1,1}$ and $Z^{1,2}$ are written in explicit form. They define the indexation of columns of matrixes constructed. Indexes of rows correspond to components of vectors $\bar{z}^1 = (z_1^1, z_2^1, z_3^1, z_4^1, z_5^1, z_6^1)$, $\bar{z}^2 = (z_1^2, z_2^2, z_3^2, z_4^2, z_5^2, z_6^2)$.

Let's construct and solve the system of equations for contact places. Notice that, in composition of subnets $Z^{1,1}$ and $Z^{1,2}$ there are such contact places as p_1 and p_5.

$$\begin{cases} z_1^1 + z_2^1 + z_4^1 - z_1^2 - z_2^2 - z_4^2 = 0, \\ z_1^1 + z_3^1 + z_5^1 - z_1^2 - z_3^2 - z_5^2 = 0. \end{cases}$$

$$(z_1^1, z_2^1, z_3^1, z_4^1, z_5^1, z_1^2, z_2^2, z_3^2, z_4^2, z_5^2, z_6^2) = \bar{y} \cdot R^1,$$

$$R^1 = \left|\begin{matrix}
0&0&0&1&1&0&1&0&0&0&0&0\\
0&0&1&1&0&0&1&0&0&0&0&0\\
0&1&1&0&0&0&1&0&0&0&0&0\\
0&1&0&0&1&0&1&0&0&0&0&0\\
1&0&0&0&0&0&0&1&1&0&0&0\\
1&0&0&0&0&0&0&1&0&0&1&0\\
1&0&0&0&0&0&0&0&1&1&0&0\\
1&0&0&0&0&0&0&0&0&1&1&0\\
0&0&0&0&1&0&0&0&1&0&0&0\\
0&0&0&0&1&0&0&0&0&0&1&0\\
0&0&0&1&0&0&0&1&0&0&0&0\\
0&0&0&1&0&0&0&0&0&1&0&0\\
0&0&1&0&0&0&0&0&1&0&0&0\\
0&0&1&0&0&0&0&0&0&0&1&0\\
0&1&0&0&0&0&0&1&0&0&0&0\\
0&1&0&0&0&0&0&0&0&1&0&0\\
1&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1
\end{matrix}\right|.$$

We construct the joint matrix G^1 out of invariants of subnets $G^{1,1}$ and $G^{1,2}$. Then we calculate the matrix $H^1 = R^1 \cdot G^1$ of basis invariants of net Z^1. The indexation of columns corresponds to vector $(x_1, ..., x_{19})$.

$$G^1 = \left|\begin{matrix}
1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&1&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
1&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0\\
0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&1&1&0&1&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&1\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0
\end{matrix}\right|,$$

$$H^1 = R^1 \cdot G^1 = \left|\begin{matrix}
1&1&0&0&1&1&1&1&1&1&1&1&0&0&1&0&0&0&0\\
1&1&0&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0\\
1&1&1&0&1&1&1&1&1&1&1&0&0&1&1&0&0&0&0\\
1&1&1&1&1&2&0&1&1&0&1&0&0&0&0&1&1&0&0\\
1&1&1&1&1&0&1&0&0&2&1&0&0&0&0&0&0&1&1\\
1&1&1&1&1&1&0&0&0&1&1&0&0&0&0&1&0&0&1\\
0&0&0&0&1&0&1&0&0&1&0&0&0&0&1&0&0&1&0\\
0&0&0&0&1&1&0&0&0&0&0&0&0&0&1&1&0&0&0\\
1&1&0&0&0&1&0&1&1&0&1&1&0&0&0&0&1&0&0\\
1&1&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0&1\\
0&0&0&1&1&0&1&0&0&1&0&0&1&0&0&0&0&1&0\\
0&0&0&1&1&1&0&0&0&0&0&0&1&0&0&1&0&0&0\\
1&1&1&0&0&1&0&1&1&0&1&0&0&1&0&0&1&0&0\\
1&1&1&0&0&0&0&0&0&1&1&0&0&1&0&0&0&0&1\\
1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0
\end{matrix}\right|.$$

Further, in analogous way we construct the invariants of entire net which is the composition of subnets Z^1 and Z^2. System of equations for contact places has the form:

$$\begin{cases}
p_{12}: & z_1^1 + z_2^1 + z_9^1 + z_{10}^1 - z_2^2 - z_{11}^2 - z_{12}^2 - z_{16}^2 = 0, \\
p_{13}: & z_2^1 + z_{11}^1 + z_{12}^1 + z_{16}^1 - z_1^2 - z_2^2 - z_9^2 - z_{10}^2 = 0, \\
p_{14}: & z_3^1 + z_{13}^1 + z_{14}^1 + z_{16}^1 - z_1^2 - z_3^2 - z_7^2 - z_8^2 = 0, \\
p_{15}: & z_1^1 + z_3^1 + z_7^1 + z_8^1 - z_3^2 - z_{13}^2 - z_{14}^2 - z_{16}^2 = 0, \\
p_{16}: & z_4^1 + z_6^1 + z_8^1 + z_{12}^1 - z_4^2 - z_9^2 - z_{13}^2 - z_{17}^2 = 0, \\
p_{17}: & z_4^1 + z_9^1 + z_{13}^1 + z_{17}^1 - z_4^2 - z_6^2 - z_8^2 - z_{12}^2 = 0, \\
p_{18}: & z_5^1 + z_7^1 + z_{11}^1 + z_{17}^1 - z_5^2 - z_6^2 - z_{10}^2 - z_{14}^2 = 0, \\
p_{19}: & z_5^1 + z_6^1 + z_{10}^1 + z_{14}^1 - z_5^2 - z_7^2 - z_{11}^2 - z_{17}^2 = 0.
\end{cases}$$

This system has 426 basis solutions constituting matrix R. After construction of joint matrix G and calculation of product $R \cdot G$, we obtain matrix H consisting of 24 basis invariants of protocol TCP model:

$$H = \left|\begin{matrix}
1&1&1&1&1&1&0&0&0&1&1&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&0&0&0&0&0&0&1&1&0&0&1&0&0&0&0&1&0&0&0&0&1&0&1&0&0&1&0\\
1&1&0&0&1&1&0&0&0&1&1&1&0&0&1&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
1&1&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0&1&0&0&0&1&1&0&1&0&0&1&0\\
1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&0&0&1&0&1&1&0&1&0&0&1&0&0&1&0&0&0&0&0&0&1&1&0&0&0&0&0\\
1&1&0&0&1&1&1&1&1&1&1&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&0&0&0&1&0&1&1&0&1&1&0&0&0&0&1&0&0&0&0&0&1&1&1&0&0&0&0&0\\
0&0&0&1&1&0&1&0&0&1&0&0&1&0&0&0&0&1&0&1&1&0&0&0&0&0&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&1&1&0&1&1&0&0&1&1&0&0&0&1&1\\
0&0&0&0&1&0&1&0&0&1&0&0&0&0&1&0&0&1&0&1&1&1&0&0&0&0&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&1&1&1&1&1&1&0&0&0&1&1\\
0&0&0&1&1&1&0&0&0&0&0&0&1&0&0&1&0&0&0&1&1&0&0&0&1&0&1&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&1&1&0&0&1&1&1&1&1&1&1\\
0&0&0&0&1&1&0&0&0&0&0&0&0&0&1&1&0&0&0&1&1&1&0&0&1&0&1&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&0&1&1&0&0&0&1&1&0&0&1&1&1&1&1&1&1&1&1&0&1&1&0&0&0&1&1\\
1&1&0&1&1&1&0&0&0&1&1&1&1&0&0&1&1&1&1&1&1&0&1&1&1&0&0&0&1&1\\
1&1&1&0&1&1&1&1&1&1&1&0&0&1&1&0&0&0&0&1&1&1&0&1&1&1&1&1&1&1\\
1&1&0&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&1&1&0&1&1&1&1&1&1&1&1\\
1&1&0&0&1&0&1&0&0&2&1&1&1&1&1&0&0&1&1&1&1&0&0&1&0&1&0&0&2&1\\
1&1&1&1&1&0&1&0&0&2&1&0&0&0&0&0&0&1&1&1&1&1&1&1&0&1&0&0&2&1\\
1&1&0&0&1&2&0&1&1&0&1&1&1&1&1&1&1&0&0&1&1&0&0&1&2&0&1&1&0&1\\
1&1&1&1&1&2&0&1&1&0&1&0&0&0&0&1&1&0&0&1&1&1&1&1&2&0&1&1&0&1
\end{matrix}\right|.$$

Since, for instance, the sum of all the invariants is a vector of naturals, model of protocol is p-invariant and, consequently, it is bounded and safe net.

In the same way, using a dual net [1] and decomposition [5], it may be shown that the model is t-invariant also. It means that net is persistent and constitutes necessary conditions for its liveness.

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