Gesellschaft
für Informatik e.V.

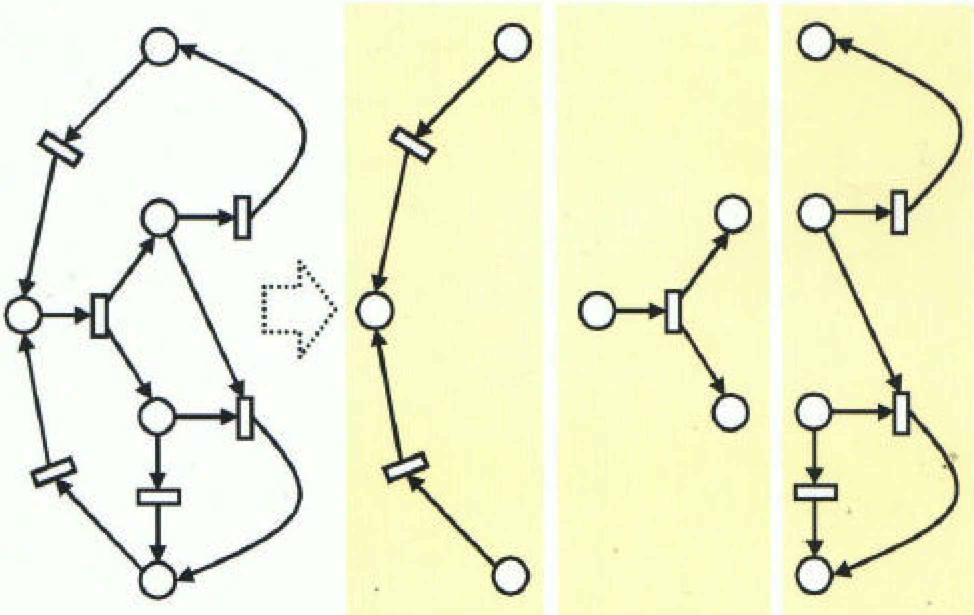
Petri Net Newsletter

Newsletter of the
Special Interest Group on

**Petri Nets and
Related System Models**

Volume 64
ISSN 0931-1084

April 2003



**Special Event: Advanced Course on Petri Nets
Eichstätt, September 15-26, 2003**

Subnets with Input and Output Places

Dmitry Zaitsev

Department of Communication Networks,
Odessa National Telecommunication Academy,
Kuznechnaya, 1, Odessa, 65029, Ukraine
E-mail: zaitsev_dmitry@mail.ru

Abstract: The aim of this paper is to introduce a concept of subnet with input and output places, especially, the specific class of the functional subnet and to construct formal methods for the decomposition of an arbitrary Petri net into subnets. The generating family of subnets is presented and grounded. An efficient polynomial algorithm of the decomposition of an arbitrary Petri net is constructed.

Introduction

There are a lot of propositions to use a union of places of elementary nets to compose a Petri net. Wide known examples can be found in [1]. Moreover it is useful to consider a Petri net as a dynamic system transforming input flow of tokens into output ones. In such case nets with input and output places are considered [2,3].

In precent work the inverse task is solved. The aim of this paper is to construct a formal method for the decomposition of an arbitrary Petri net into subnets with input and output places. A polynomial algorithm of the decomposition is obtained. The result of the concrete Petri net decomposition is presented at the Cover Picture.

Note that this little paper is in essence a brief overview of [4], which contains all the proofs, the analysis of another approaches to decomposition and some additional concepts.

Basic concepts

Petri net is a triple $N=(P,T,F)$, where P is the finite set of places, T is the finite set of transitions; the flow relation $F \subseteq P\times T \cup T\times P$ defines a set of arcs connecting places and transitions. Hence, a Petri net is bipartite directed graph [1]. One part of nodes consists of places, another part consists of transitions. In this work we do not introduce a concepts of token and dynamics of the net. Only structural properties will be investigated. All results will be obtained for the class of bipartite directed graphs; therefore they are applied to various classes of Petri nets used graph N.

We introduce the special notations for the sets of input, output and incident nodes of the place: ${}^\bullet p=\{t \mid \exists(t,p)\in F\}$, $p^\bullet=\{t \mid \exists(p,t\}\in F\}$, ${}^\bullet p^\bullet={}^\bullet p \cup p^\bullet$.

Similarly we may define the sets of input, output and incident nodes of the transition and of an arbitrary subset of places (transitions).

A ***net with input and output places*** is a triple $Z=(N,X,Y)$, where N is Petri net, $X\subseteq P$ - input places, $Y\subseteq P$ - output places and the sets of input and output places do not intersect $X\cap Y=\varnothing$. Places from the set of $Q=P\setminus(X\cup Y)$ are called ***internal***.

During investigation of such nets the external flow of tokens is directed to the input places and the result of the net behavior is observed in the output places [2,3].

Input and output places are named the ***contact*** ones. Dependent on the constraints imposed on the set of arcs of contact places, there are various definitions [2,3] of such nets. In this work we shall consider the strictest class so called functional nets [3].

A ***functional net*** is a net with input and output places such that the input places do not have input arcs and output places do not have output arcs: $\forall p \in X: {}^{\bullet}p = \varnothing$, $\forall p \in Y: p^{\bullet} = \varnothing$. Input places are the sources and output places are the drains in the terms of the graph theory [1]. We shall denote the functional net as $Z = (X, Q, Y, T, F)$, with respect to correspondent elements of Petri net *N*. In the general case the set of contact places of a functional net may be empty.

<u>**Statement 1**</u>. An arbitrary Petri net *N* may be considered as the functional net where the set *X* consists of the sources and the set *Y* consists of the drains of the net *N*.

Further, do not restricting the generality, we shall consider only functional nets allowing the empty sets of contact places.

A Petri net $N' = (P', T', F')$ is ***subnet*** of *N*, if $P' \subseteq P, T' \subseteq T, F' \subseteq F$.

A net $N' = (P', T', F')$ is called a ***subnet*** of *N* **generated by the specified set of nodes** $B(P', T')$ if N' is a subnet of *N* and F' contain all the arcs connecting the nodes P', T' in the source net:

$$F' = \{(p,t) \mid p \in P', t \in T', (p,t) \in F\} \bigcup \{(t,p) \mid p \in P', t \in T', (t,p) \in F\}.$$

A ***subnet generated by the specified set of transitions*** $B(T')$ is a subnet $B(P', T')$, such that $P' = \{p \mid p \in P, \exists t \in T' : (t,p) \in F \vee (p,t) \in F\}$. In other words, together with the transitions from T' subnet contains all the incident places and is generated by these nodes.

Further we shall consider mainly all the arcs connecting the specified nodes in the source net; that is we shall consider subnets generated by the set of nodes. Therefore for brevity we shall omit the flow relation implying the source relation *F*.

A functional net $Z = (N', X, Y)$ is a ***functional subnet*** of the net *N* and denoted by $Z \succ N$, iff N' is the subnet of *N* and moreover Z is connected with the residuary part of the net only by the arcs incident to contact places so that input places may have only input arcs and output places may have only output arcs. Hence:

$$\forall p \in X : \{(p,t) \mid t \in T \setminus T'\} = \varnothing, \ \forall p \in Y : \{(t,p) \mid t \in T \setminus T'\} = \varnothing,$$

$$\forall p \in Q : \{(p,t) \mid t \in T \setminus T'\} = \varnothing \wedge \{(t,p) \mid t \in T \setminus T'\} = \varnothing.$$

<u>**Statement 2**</u>. A functional subnet is generated by the set of own transitions.

So, the set of transitions defines the functional subnet uniquely. If we consider the constraints of the arcs for the contact places we shall conclude that the residuary part of the source net is the functional subnet also. At that the source net may be obtained by the way the union of subnets by means of merging of the contact places of opposite classes: input with output and output with input.

The ***difference*** of the source Petri net *N* and it's functional subnet Z' is the subnet $Z'' = N - Z'$ where $Z'' = (Y, P \setminus (X \bigcup Y \bigcup Q), X, T \setminus T')$.

<u>**Statement 3**</u>. If $Z' \succ N$, then $N - Z' \succ N$.

A functional subnet $Z' \succ N$ is a ***minimal*** iff it does not contain any another functional subnet of Petri net *N*.

Properties of the functional subnets

Theorem 1. The sets of transitions of two arbitrary minimal functional subnets Z' and Z'' of Petri net N do not intersect.

Conclusion 1. The sets of internal places of two arbitrary minimal functional subnets Z' and Z'' do not intersect.

Conclusion 2. The set of minimal functional nets $\Im = \{Z^j\}$, $Z^j \succ N$ defines the partition of the set T into nonintersecting subsets T^j so $T = \bigcup_j T^j$, $T^j \bigcap T^k = \varnothing$, $j \neq k$.

It is necessary to be mentioned that in the general case the minimal subnet does not imply a not great quantity of nodes but suppose that the subnet is not to be divided into an internal subnets.

Theorem 2. Any functional subnet Z' of an arbitrary Petri net N is the sum (union) of the finite number of the minimal functional subnets.

In Fig. 1 the functional subnet that is the sum of two minimal functional subnets shown at Cover Picture is presented.

Conclusion. The partition of the set T defined by the set of minimal functional subnets is the generating family of the set of the functional subnets of Petri net N.

The goal of this work is to construct an efficient algorithm for the search of such generating family for a given Petri net.

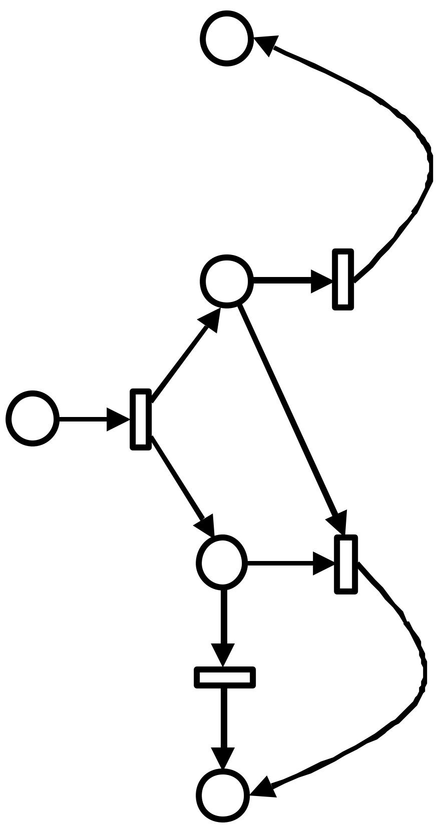


Fig. 1. Functional subnet as the sum of two minimal subnets.

Algorithm of decomposition

Subnet $Z = B(R) = (X, Q, Y, R)$ of Petri net *N* is a ***complete*** in *N*, iff conditions $X^\bullet \subseteq R$, ${}^\bullet Y \subseteq R$, ${}^\bullet Q^\bullet \subseteq R$ hold true in *N*.

Algorithm 1:

Step 0. Choose an arbitrary transition $t \in T$ of the net *N* and include it in the set of selected transitions $R := \{t\}$.

Step 1. Construct the subnet *Z* generated by the set *R*: $Z = B(R) = (X, Q, Y, R)$.

Step 2. If *Z* is the complete in *N*, then *Z* is the sought subnet. Stop.

Step 3. Construct the set of the absorbed transitions:

$$S = \{t \mid t \in X^\bullet \wedge t \notin R \vee t \in {}^\bullet Y \wedge t \notin R \vee t \in {}^\bullet Q^\bullet \wedge t \notin R\}.$$

Step 4. Assign $R := R \bigcup S$ and go to Step 1.

Theorem 4. Subnet *Z* is the complete in Petri net *N* iff it is the functional subnet of *N*.

Theorem 5. Subnet *Z* constructed by Algorithm 1 is the minimal functional subnet of Petri net *N*.

Thus, Algorithm 1 construct a minimal functional subnet *Z* of Petri net *N*. Let us assume $i := 1$ and $Z^i := Z$. Then we assign $N := N - Z$ and repeat execution of Algorithm 1 in the case if the set *T* is not empty. Continuing in such a manner and choosing $i := i + 1$, we construct the set of minimal functional subnets $Z^1, Z^2, ..., Z^k$ of Petri net *N* representing the desired partition of the source net.

The result of execution of the algorithm is shown at the Cover Picture. The picture represents the source net and the set of minimal subnets as the result of the decomposition.

Theorem 6. Algorithm 1 has the polynomial complexity $o(n^3)$, where *n* is the number of nodes of the net.

Note that the algorithm processes each transition of the net once; the higher degree of the polynomial in the estimation of the complexity is defined in the general case by the number of arcs connecting the nodes of the net. An upper bound of them in the complexity estimation is chosen as n^2. The practical investigation of the dozens of real models using program realization of Algorithm 1 showed that models of real systems have no high density of the arcs; in most cases it does not exceed a constant *c* for every node of the net. In that case the complexity of the algorithm is linear.

References

1. Peterson J. Petri Net Theory and the Modelling of Systems, Prentice Hall, 1981.
2. Ichikawa A., Yokoyama K., Kurogi S. Reachability and control of discrete event systems represented by conflict-free Petri nets // Proc. of ISCAS 85, pp. 487-490.
3. Zaitsev D.A., Sleptsov A.I. State Equations and Equivalent Transformations of Timed Petri Nets // Cybernetics and System Analysis, Vol. 33. ¹ 5, 1998, pp. 659-672.
4. Zaitsev D.A. Decomposition of Petri Nets // Kibernetika i sistemny analiz, 2002, ISSN 0024-1274. In press.